Testing under Information Manipulation

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Abstract

A decision-maker commits to a standard of evidence to discourage low-type agents and encourage high-type agents to improve their signal distribution. If the effect of commitment on low-type agents dominates its effect on high-type agents, optimal commitment standards are *confirmative*: for large priors that the agent's type is low, the optimal standard is *harsh*; i.e., it requires more favorable evidence than ex-post optimal tests to choose the agent's preferred action. Similarly, for large priors that the agent's type is high, the optimal standard is *soft*. If the effect of commitment on high-type agents dominates its effect on low-type agents, these results reverse and optimal commitment standards are *conservative* (cf. Li (2001)). Commitment is Pareto improving for some priors. A revelation mechanism Pareto dominates commitment for large priors that the type is low, and is generically preferred by the decision-maker over simple commitment to a standard.

Keywords: Information manipulation, Commitment, Ex-post inefficiency, Confirmativism, Conservatism, Standard of Evidence.

JEL Classification: C72; D82; D86.

1 Introduction

In the wake of the replicability crisis, the scientific community has come to realize that methodologies and practices followed for decades, or even centuries, are not immune to malign incentives and research misconduct in particular.[1] A number of alternative solutions have been put forth; for instance, recently, the Ministry of

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[1]Di Tillio et al. (2017) provide a historical account of the development of experimental methods.

Science and Technology and courts in China have moved towards hard penalties for scientific misconduct —the most extreme even considering the death penalty.[2] However, in many fields, in practice it is nearly impossible to detect research misconduct and even more difficult to prove it, rendering potential punishment virtually ineffective.[3] The logic of the problem is not exclusive to research misconduct. In civil litigation, evidence tampering is pervasive. Sanchirico (2004) points out that *"according to many judges and practitioners[,]...documents that should be produced in response to a discovery request are regularly shredded, altered, or suppressed."* Another example is *test-defensiveness* in psychology. According to Butcher (2002): *"When taking psychological tests at pre-employment, pilots who have personality problems and other mental health symptoms can respond in a way to 'mask' those problems."*[4]

We explore alternative ways to disincentivize research misconduct, evidence tampering, test-defensiveness, and other forms of evidence manipulation, by altering the design of the decision process.[5] We consider a decision maker facing a binary decision problem, and an agent who prefers one of the actions, regardless of her type. Before choosing an action, the decision maker observes evidence that is partially informative about the agent's type: an editor decides whether to publish a manuscript, a judge decides whether to acquit a defendant, and a manager decides whether to hire a pilot applying to a job in a commercial airline. Regardless of her type, the agent may exert hidden efforts to alter the evidence in order to improve the chances of a favorable decision.[6] Since the decision maker's and high type agents' incentives are aligned, we refer to their effort as *information generation*. In contrast, we refer to the effort of low type agents as *information undermining*. Our analysis considers both forms of *information manipulation*.

First, we analyze a simple model of *commitment* to a standard of evidence. In order to discourage information undermining and incentivize information generation, the resulting standards under commitment differ from optimal statistical decision-making. We characterize the direction of the deviations from ex-post opti-

[2]Source: `https://www.statnews.com/2017/06/23/china-death-penalty-research-fraud/` (STATNEWS June 23, 2017).

[3]Although punishment is not completely ineffective in all fields, within several of them, uncovering misconduct may not be practical (see, e.g., Fanelli (2009)).

[4]Airlines' screening of pilots was subject to intensive scrutinity in 2015, in the aftermath of a Germanwings plane crash in the Alps, believed to be deliberately caused by the pilot.

[5]In the absence of concerns about collateral effects on the agent's incentives, such decisions are analyzed as standard statistical decision problems (see, e.g., Neyman and Pearson (1933), Karlin and Rubin (1956), and DeGroot (2005)).

[6]For instance, Butcher (1994) suggests that high average performance of pilots in psychological tests can be explained by fit pilots' attempts to display "overly favorable response patterns."

mality in terms of two interacting factors: the manipulation effect that is dominant (undermining or generation), and the prior probability that the agent's type is low (Propositions 1, 3, 4, and 5). Second, we analyze a revelation mechanism for this problem.

Commitment improves the expected payoff of the decision maker and, when it involves lowering the standard, it leads to outcomes that are Pareto superior to those without commitment (Corollaries 4 and 5). In turn, the optimal mechanism Pareto dominates commitment to a standard for relatively high priors that the type is low (Propositions 6 and 7).

We characterize when optimal standards of evidence are *harsh* or *soft*; i.e., require, respectively, more or less favorable evidence than optimal statistical tests for choosing the agent's preferred action.[7] In the pure (or dominating) information undermining setting, for large prior probabilities that the type is high, the ex-ante optimal standard is soft; and, for large prior probabilities that the type is low, the ex-ante optimal standard is harsh (Propositions 1 and 3). Soft standards help decision making if low type agents' effort is a strategic complement of the standard (Lemma 2). In turn, under the MLRP assumption, strategic complementarity develops for the low standards that arise in equilibrium, when the agent's type is likely to be high (Lemmata 1 and 3). An analogous reasoning reveals why harsh standards are ex-ante optimal when the agent's type is likely to be low.

A qualitative implication of this result is that, under pure (or dominating) information undermining, optimal standards tend to be *confirmative*; i.e., they often favor the optimal decision based on prior information only (Corollary 1).[8] Confirmative standards —soft standards for new drugs with good prior prospects in particular— are consistent with recent findings on the approval of new drugs for which the FDA has granted a Breakthrough-Drug Designation. This designation is given based on preliminary evidence to drugs that could provide a substantial improvement to what is available on the market. A number of drugs that received this designation, however, were approved by the FDA, despite subsequent trials showing limited efficacy. Lowering the standard may benefit decision making by discouraging information undermining (see Section 8 for further discussion).

In contrast, in the pure (or dominating) information generation model, *conservative* standards —i.e., standards adjusted in the opposite direction to the optimal decision based on prior information only— are ex-ante optimal more often (Propo-

[7]Commitment does not always lead to ex-post inefficiencies (see, e.g., Li and Suen (2004)).

[8]Indeed, we find families of distributions such that all standard's deviations from ex-post optimality are confirmative for all agent's hidden effort costs (Proposition 2).

sitions 4 and 5, and Corollary 2). Li (2001) shows that conservative standards also arise in information aggregation models with costly information acquisition. Both Li (2001) and this paper highlight that the quality of information is endogenous to decision-making. While he focuses on mitigating free-riding by committee members, we focus on discouraging (encouraging) effort by low (high) type agents —who, in Li's model, are non-strategic. In his setup, committee members' effort reduces the variance of the signal; in ours, agents' effort shifts probability mass to the right.

Finally, we consider revelation mechanisms without transfers: given a reported type, the decision maker sets probabilities of outright acceptance, outright rejection, or taking a test. In the optimal mechanism, only candidates reporting a high type are tested. We provide conditions such that, for large prior probabilities that the agent's type is low, the agent prefers the optimal mechanism over simple commitment to a standard, and the opposite when those probabilities are small. Since in the optimal mechanism the low type is not tested, the decision maker extracts the manipulation cost and obtains higher expected payoffs than when he commits to a standard.

Related literature. The work of Li (2001) discussed above is the closest antecedent to our work. Our setup also relates to the model studied by Frankel and Kartik (2019a), in which, agents maximize the expected market-valuation of their quality, determined directly by their "natural" type and indirectly by their gaming-ability type. While their focus is on the equilibrium informativeness about type dimensions, ours is on the decision-maker's management of information manipulation.[9]

Three concurrent working papers analyze related problems: Cunningham and Moreno de Barreda (2019) show that costly signal-jamming improves a sender's probability of persuading the receiver in a model with uniformly distributed types.[10] In their model, signal-jamming makes the receiver worse-off and commitment always leads to harsh standards. In contrast, in our model, information manipulation may be dominated by information generation making the decision-maker (the receiver) better-off; and, under pure or dominant information undermining or generation, commitment *always* leads to soft standards for a range of priors.

[9] Similarly, Ederer et al. (2018) analyze how "opaque" contracts help a principal incentivize an agent to exert balanced efforts between tasks. In contrast, in our paper, standards' distortions aim to discourage (encourage) the low (high) type agent to exert effort.

[10] Also related is the literature on "influence activities," (see, e.g., Meyer et al. (1992)). Driven by rent-seeking, influence activities usually lead to resource misallocation within organizations. Recent work, however, has studied the benefits of influence activities, such as helping information transmission (Choe and Park (2017)) or incentivizing information acquisition (Laux (2008)).

Frankel and Kartik (2019b) show how under-response to information may help a receiver to discourage data manipulation, and Ball (2020) analyzes how the receiver's commitment problem can be mitigated by introducing an intermediary who distorts and coarsens primitive information. While these papers and ours exhibit moral hazard, our analysis focuses on how to tailor commitment to the specific manipulation faced by the decision maker and how to take advantage of revelation mechanisms to address manipulation.

Taylor and Yildirim (2011) consider how evidence standards play a dual role: as a selection criterion and as a tool to incentivize an agent whose effort increases the probability that her type is high. Although their analysis focuses on comparing blind versus informed reviews, they also consider a model with commitment to a standard in which the principal observes the agent's ability but not her type, and accordingly sets different standards. Optimal standards are harsh (soft) for agents with low-cost (high-cost) effort, resembling our findings for the pure information generation model. The driving forces behind their findings and ours are different, however, as in our model, the agent has private information and effort is purely wasteful (it does not affect the probability that the agent's type is high).

Our paper contributes to a growing literature on research practices and economic incentives.[11] Di Tillio et al. (2017, 2018) study how scientists' persuasion bias affects the informativeness of experiments, explicitly considering the probabilistic structure of sample selection. In contrast, our analysis abstracts from the specifics of information manipulation —yet, to illustrate, in the Appendix we show how specific data manipulation processes, arising from hidden sample design and data disposal, fit within our setup (or trivial generalizations). Our framework is very different from persuasion models *a la* Gentzkow and Kamenica (2011):[12] (i) in our model, there is asymmetric information, because the sender (the agent) knows her type; (ii) our model has moral hazard: the receiver (the decision maker) does not observe the signal distribution chosen by the sender; (iii) the sender is restricted to choose within a set of signal distributions, and it is costly to choose more favorable signal distributions; and (iv) the receiver affects the sender's incentives by committing to a standard.

The distinctive feature of our model, in comparison to classical statistical problems, is the presence of moral hazard. Our mechanism design approach, however, is rather shaped by information asymmetry: in the optimal revelation mechanism,

[11] See, e.g., Chassang et al. (2012), Tetenov (2016), Henry and Ottaviani (2019), Di Tillio et al. (2017, 2018), Herresthal (2018), McClellan (2020), and references therein.

[12] See, e.g., Kolotilin (2015), Hedlund (2017). Within this literature, Perez-Richet and Skreta (2018) is the closest reference, as they consider manipulation within a Bayesian persuasion setup.

the menu offered by the decision-maker has features resembling a discrete-type version of Mussa and Rosen (1978) price discrimination model.[13] Our setup, however, is different, because the manager has aligned (opposite) interests with the high (low) type agent.[14]

2 The Model

A *manager* (he) decides whether to *hire* or *reject* a *candidate* (she). He prefers hiring the candidate if she is *fit* and rejecting her if she is *unfit*. The candidate's fitness, however, is not observable to the manager, and the candidate is fit with a prior probability strictly between 0 and 1, and unfit otherwise. The prior unfitness odds, i.e., the prior probability that the candidate is unfit divided by the prior probability that the candidate is fit, are denoted by κ.

The manager is risk-neutral and minimizes expected losses. Without loss of generality, the manager's losses due to hiring unfit candidates and rejecting fit candidates are normalized to 1.[15] For $\kappa < (>)1$, if the manager were to make his decision based on prior information only, he would choose hiring (rejection). Throughout the paper we refer to κ simply as the *prior*. A useful interpretation for the reader to keep in mind is that κ corresponds to a measure of the manager's relative expected loss from hiring given the prior information.

2.1 Evidence

The manager runs a test to obtain further evidence on the candidate's fitness. The result of the test is the realization of a signal $z \in [0, 1]$. The distribution of the signal is determined by the candidate's *readiness* for the test, $\theta \in [\underline{\theta}, \overline{\theta}] =: \Theta$. The distribution and density functions of a candidate's signal with readiness θ are denoted by $F(\cdot, \theta)$ and $f(\cdot, \theta)$, respectively. Thus, the domain of F is $D := [0, 1] \times \Theta$ and its interior is denoted by D°; similarly, the interior of Θ is denoted by Θ°.

[13] For instance, the low type agent is indifferent between reporting her true type or lying, whereas the high type agent strictly prefers reporting her type.

[14] Our paper also relates to the literature on optimal evidentiary legal standards to induce adequate behavior (see, e.g., Demougin and Fluet (2008), Ganuza et al. (2015), Gerlach (2013), Kaplow (2011), Sanchirico (2012)). The informative role of evidentiary standards has received little attention in this literature. Stephenson (2008) analyzes the effect of standards on the research effort of agencies seeking court approvals, and Mungan and Samuel (2019) show that harsh standards deter crime when guilty agents mimic innocent ones. Their work, however, has no counterpart to the characterizations of harsh versus soft standards, or confirmativism versus conservatism, provided here.

[15] An increase in the relative weight of hiring the unfit candidate over the weight of rejecting the fit candidate has the same effect as an increase in κ.

Assumption F.1 *The distribution F is atomless and thrice continuously differentiable on D°, and $f(z,\theta) > 0$ for all $(z,\theta) \in (0,1) \times \Theta$.*

The partial derivatives of F are defined on the interior of D, and extended to the boundary points in the usual way, taking the limits —which we assume to exist throughout.[16] We also assume that the density is log-supermodular:

Assumption F.2 *The distribution F satisfies*

$$\frac{\partial^2 \ln f(z,\theta)}{\partial\theta\partial z} > 0 \tag{1}$$

for all $(z,\theta) \in D^\circ$.

Assumption F.2 implies the strict Monotone Likelihood Ratio Property (MLRP): if $\theta' > \theta$, then $\frac{f(\cdot,\theta')}{f(\cdot,\theta)}$ is strictly increasing, and, since MLRP implies strict first-order stochastic dominance (FOSD), $F(z,\theta) > F(z,\theta')$ for all $z \in (0,1)$.

All the proofs are in the Appendix.

2.2 Evidence Standards

Throughout the paper, we only consider readiness pairs $\boldsymbol{\theta} := (\theta_u, \theta_q) \in \boldsymbol{\Theta} := \{\boldsymbol{\theta} \in \Theta^2 : \theta_u < \theta_q\}$, where θ_u and θ_q are the readiness of the unfit and fit candidates, repectively. By MLRP, the manager's best response to any $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ is an acceptance standard; i.e., a "threshold" strategy (s) such that the manager hires the candidate if $z \geq s$ and rejects her if $z < s$ for some $s \in [0,1]$. For any standard s, and readiness profile (θ_u, θ_q), the probabilities of wrongful rejection and wrongful hiring are, respectively, $F(s,\theta_q)$ times the prior probability that the candidate is fit and $(1 - F(s,\theta_u))$ times the prior probability that the candidate is unfit. Thus, the manager's expected loss is an affine transformation of[17]

$$V(s,\boldsymbol{\theta}) = F(s,\theta_q) - \kappa F(s,\theta_u) \tag{2}$$

for all $(s,\boldsymbol{\theta}) \in [0,1] \times \boldsymbol{\Theta}$. We will often explicitly indicate the dependence of V on the prior κ, writing $V(s,\boldsymbol{\theta};\kappa)$ instead of $V(s,\boldsymbol{\theta})$.

In a particular case of this problem, the candidate has a "natural" readiness for the test, $\underline{\theta}$ if she is unfit, and $\underline{\theta}_q > \underline{\theta}$ if she is fit. The problem solved by the manager facing a candidate with her natural readiness is called *classical statistical problem.*

[16] Assumption F.1 guarantees that these derivatives are real functions on D°, but since they do not need to be bounded, they may be infinite on the boundary points.

[17] The manager's expected loss is $(V + \kappa)$ times the prior probability that the candidate is fit.

The manager's expected loss in this problem is $V(s, \underline{\theta}, \underline{\theta}_q) = F(s, \underline{\theta}_q) - \kappa F(s, \underline{\theta})$, for all $s \in [0,1]$. Let $\underline{\kappa} := g(0, \underline{\theta}, \underline{\theta}_q)$ ($\overline{\kappa} := g(1, \underline{\theta}, \underline{\theta}_q)$) be the largest (smallest) prior κ such that the candidate is outright hired (rejected) in the classical statistical problem. Accordingly, we say that a classical statistical problem is *testworthy* if $\kappa \in (\underline{\kappa}, \overline{\kappa})$. MLRP implies that $\underline{\kappa} < 1 < \overline{\kappa}$. [18]

2.3 Information manipulation

By exerting effort, the candidate increases her readiness —formally, we define *effort* by the unfit and fit candidates as $\theta_u - \underline{\theta}$ and $\theta_q - \underline{\theta}_q$, respectively. The unfit candidate's action set is Θ, and the cost of increasing her readiness is given by $C_u : \Theta \to \mathbb{R}_{\geq 0}$. The fit candidate's cost function is $C_q(\cdot; \underline{\theta}_q) : \Theta_q \to \mathbb{R}_{\geq 0}$, with $\Theta_q = \{\theta \in \Theta : \theta \geq \underline{\theta}_q\}$. To avoid notation cluttering, we often are not explicit about the fit candidate's cost dependence on $\underline{\theta}_q$, and simply write $C_q(\cdot)$ (and similarly for its derivatives).

The candidate is risk-neutral and her loss from not being hired is normalized to 1. Given a standard $s \in [0,1]$, the unfit and fit candidates' expected losses are

$$U_i(s, \theta_i) := F(s, \theta_i) + C_i(\theta_i), \tag{3}$$

for $i = u, q$, respectively, for all $\theta_u \in \Theta$ and $\theta_q \in \Theta_q$.

Assumption C. 1 *The cost functions C_u and C_q (i) are twice continuously differentiable, (ii) satisfy $C_u(\underline{\theta}) = C_u'(\underline{\theta}) = 0$, and for all $\underline{\theta}_q \in \Theta^\circ$, $C_q(\underline{\theta}_q; \underline{\theta}_q) = C_q'(\underline{\theta}_q; \underline{\theta}_q) = 0$, (iii) $C_i'' > -\frac{\partial^2 F(s,\cdot)}{\partial \theta^2}$ over the interior of their respective domains, for all $s \in (0,1)$ and $i = u, q$; (iv) $C_i'(\overline{\theta}) > -F_\theta(s, \overline{\theta})$ for all $s \in (0,1)$ and $i = u, q$; and (v) $C_q'(\theta; \underline{\theta}_q) < C_u'(\theta)$ for all $\theta \in (\underline{\theta}_q, \overline{\theta})$.*[19]

Part (iii) of C.1 is a mild condition[20] guaranteeing convexity of the candidate's loss function; (iv) implies that $\overline{\theta}$ is not an optimal choice for the candidate; and (v) imposes that the unfit candidate is less ready for the test than the fit candidate.

Manipulation processes. Several manipulation processes can be analyzed with our model or trivially modified versions. In the Appendix, we provide an example

[18] By Remark 3 and continuity of $\frac{\partial f(s,\theta)}{\partial \theta}$, $\underline{\kappa}$ ($\overline{\kappa}$) is weakly decreasing (weakly increasing) in $\underline{\theta}_q$. Therefore, the range of priors for which the classical statistical problem is testworthy gets larger as $\underline{\theta}_q$ increases.

[19] The derivatives of functions of one variable are denoted by a prime or $\frac{d}{dx}$, where x is the variable.

[20] For instance, (iii) is automatically satisfied if cost functions are convex ($C_u'' > 0$ and $C_q''(\cdot; \underline{\theta}_q) > 0$ over the interior of their respective domains) and F satisfies Convexity of the Distribution Function (CDFC).

of sample cherry-picking and outline an example of data selection.

2.4 Commitment to Standards

We consider a dynamic game, denoted by Γ_1, in which, (i) first, Nature chooses the candidate's type (fit or unfit) and reveals it to the candidate; (ii) the manager (without observing the candidate's type) commits to a standard $s \in [0,1]$. Then, in stage (iii), having observed the standard chosen by the manager, the candidate chooses readiness θ. (iv) Nature chooses a signal realization $z \in [0,1]$ according to $F(\cdot, \theta)$, and the candidate is hired if and only if $z \geq s$.[21]

3 Analysis of Commitment

3.1 Strategic Complementarity/Substituibility of Readiness

Given a standard s, the optimal readiness, denoted by $\theta_i^*(s)$, is a minimizer of $U_i(s, \cdot)$ for $i = u, q$. Since $F(0, \theta) = 0$ and $F(1, \theta) = 1$ for all $\theta \in \Theta$, $\theta_u^*(0) = \theta_u^*(1) = \underline{\theta}$ and $\theta_q^*(0) = \theta_q^*(1) = \underline{\theta}_q$. Furthermore, by Assumptions F.1-F.2 and C.1, $\theta_i^*(s)$ is an interior solution and satisfies

$$C_i'(\theta_i^*(s)) = -F_\theta(s, \theta_i^*(s)), \tag{4}$$

for all $s \in (0,1)$ and $i = q, u$; and $\theta_q^* > \theta_u^*$.

Readiness is a strategic complement (substitute) of the standard at (s, θ) when (s, θ) is located in the submodular (supermodular) region of F:

$$\frac{d\theta_i^*(s)}{ds} = -\frac{\partial f(s, \theta_i^*(s))}{\partial \theta} \left(C_i''(\theta_i^*(s)) + \frac{\partial^2 F(s, \theta_i^*(s))}{\partial \theta^2} \right)^{-1} \tag{5}$$

for all $s \in (0,1)$ and $i = q, u$. By part (iii) of Assumption C.1, the sign of the effect of the standard on the candidate's optimal readiness is determined by the sign of $\frac{\partial f(s, \theta_i^*(s))}{\partial \theta}$; i.e., by whether F is sub or supermodular at $(s, \theta_i^*(s))$.

Remark 1 *Assume F.1-F.2 and C.1. Readiness is a strategic complement (substitute) of the standard at* $s \in [0,1]$ *(i.e.,* $\frac{d\theta_i^*(s)}{ds} > (<) 0$*) if and only if* $(s, \theta_i^*(s))$ *is located in the submodular (supermodular) region of* F*, for* $i = q, u$.

[21] Implicitly, here we assume that agents are capable –as they are in many applications– to "sabotage" the result of the test. This obligates the manager to define hiring and rejection sets using a standard, preventing him from (potentially) taking advantage of other ways to determine these sets. This might explain why standards are widespread in practical decision-making.

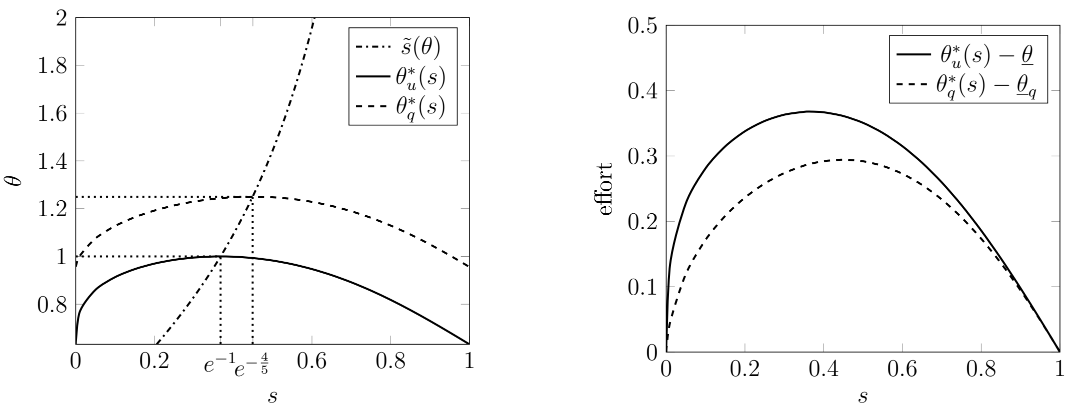

Figure 1: **Example 1.** Left panel: candidates' best responses and $\widetilde{s}$. Right panel: candidates' effort for each standard.

We now provide the first of three lemmas leading to Propositions 1 and 4, which are key results of the paper.

Lemma 1 *Assume F.1-F.2 and C.1. For $i = u, q$, there exists $\hat{s}_i \in (0,1)$ such that*

$$\frac{\partial f(s,\theta_i^*(s))}{\partial\theta}\begin{cases} <0 & \text{if } 0 \leq s < \hat{s}_i \\ =0 & \text{if } s = \hat{s}_i \\ >0 & \text{if } \hat{s}_i < s \leq 1. \end{cases} \tag{6}$$

We call this property *single modularity-switch*: there exists a cut-off $\hat{s}_i$, which we call the *modularity-switch point*, such that for standards smaller (greater) than $\hat{s}_i$, F is submodular (supermodular) at $(s, \theta_i^*(s))$. The proof relies on the fact that we can separate the submodular regions of the domain of F from the supermodular regions, using the function $\widetilde{s}$ defined in Remark 3. Lemma 1 and (5) reveal that θ_i^* is strictly increasing over $[0, \hat{s}_i]$ and strictly decreasing over $[\hat{s}_i, 1]$ for $i = u, q$.

Example 1 *Consider $F(z,\theta) = z^\theta$ for all $(z,\theta) \in [0,1] \times \Theta$ with $\Theta = [1-\mathrm{e}^{-1}, 2]$. For this function, $\widetilde{s}(\theta) = \mathrm{e}^{-\frac{1}{\theta}}$ for all $\theta < \overline{\theta}$.*

The cost functions are $C_u(\theta) = \frac{1}{2}(\theta - \underline{\theta})^2$ for all $\theta \in \Theta$ and $C_q(\theta) = \frac{1}{2}(\theta - \underline{\theta}_q)^2$ for all $\theta \in \Theta_q$. The unfit candidate's optimal readiness $\theta_u^*(s)$ is the root of $-s^{\theta_u^*(s)} \ln s = \theta_u^*(s) - \underline{\theta}$ for all $s \in (0,1)$. The only duplet $(s,\theta) \in [0,1] \times \Theta$ in the intersection of the graphs of $\widetilde{s}$ and θ_u^* is $(\hat{s}_u, \theta_u^*(\hat{s}_u)) = (\mathrm{e}^{-1}, 1)$. Similarly, if we set $\underline{\theta}_q = \frac{5}{4} - \frac{4}{5\mathrm{e}}$, we have $(\hat{s}_q, \theta_q^*(\hat{s}_q)) = (\mathrm{e}^{-\frac{4}{5}}, \frac{5}{4})$. The left panel of Figure 1 shows $\widetilde{s}$, θ_u^*, and θ_q^*, and the right panel displays the corresponding efforts.

The manager's and candidate's expected losses are given by (2) and (3), respectively, and their preferences, including κ, $\underline{\theta}_q$, F, C_u, and C_q, are common knowledge. The unfit and fit candidates' sets of strategies, denoted by $\Theta^{[0,1]}$ and $\Theta_q^{[0,1]}$, respectively, are set of functions mapping standards to readiness.

Our analysis focuses on *Subgame Perfect Nash Equilibria (SPNE) in pure strategies*; i.e., triplets $\left(s_P^*, \theta_u^*, \theta_q^*\right) \in [0,1]\times\Theta^{[0,1]}\times\Theta_q^{[0,1]}$ such that $V(s_P^*, \theta_u^*(s_P^*), \theta_q^*(s_P^*)) \leq V(s, \theta_u^*(s), \theta_q^*(s))$ for all $s \in [0,1]$, and $U_u(s, \theta_u^*(s)) \leq U_u(s, \theta)$ for all $\theta \in \Theta$ and $U_q(s, \theta_q^*(s)) \leq U_q(s, \theta)$ for all $\theta \in \Theta_q$, for all $s \in [0,1]$.[22] It is not difficult to show that, under Assumptions F.1, F.2, and C.1, a SPNE always exists.

3.2 Harsh and Soft Standards

The manager's expected loss as a function of the standard is given by

$$\mathcal{V}(s) := V(s, \theta_u^*(s), \theta_q^*(s)) = F(s, \theta_q^*(s)) - \kappa F(s, \theta_u^*(s)),$$

with

$$\frac{d\mathcal{V}(s)}{ds} = f(s, \theta_q^*(s)) + F_\theta(s, \theta_q^*(s))\frac{d\theta_q^*(s)}{ds} - \kappa f(s, \theta_u^*(s)) - \kappa F_\theta(s, \theta_u^*(s))\frac{d\theta_u^*(s)}{ds}, \quad (7)$$

for all $s \in (0,1)$. Using the total derivatives of the distribution of the signal for each type, $d_i(s) := \frac{dF(s,\theta_i^*(s))}{ds} = f(s, \theta_i^*(s)) + F_\theta(s, \theta_i^*(s))\frac{d\theta_i^*(s)}{ds}$ for $i = u, q$ and $s \in [0,1]$, we define the *pseudo likelihood ratio function* v, with

$$v(s) := \frac{d_q(s)}{d_u(s)},$$

for all $s \in \{z \in (0,1) : d_u(z) \neq 0\} \cup \{0,1\}$. Thus, for all SPNE such that $s_P^* \in (0,1)$, we have $v(s_P^*) = \kappa$.

In setting the standard, the manager takes into account the candidate's incentives to exert effort —the second and fourth terms on the right-hand side of equation (7). Thus, equilibrium standards, in general, are not ex-post efficient.

Definition 1 *Let $(s_P^*, \theta_u^*, \theta_q^*)$ be a SPNE of Γ_1. The equilibrium standard is* soft (harsh) *if $s_P^* < (>)\, s^*\left(\theta_u^*(s_P^*), \theta_q^*(s_P^*); \kappa\right)$.*

Upon observing the signal, at the margin, a harsh manager rejects a candidate even if the expected loss from hiring is strictly less than the expected loss from rejection. Similarly, at the margin, a soft manager hires a candidate even if the

[22]Strictly speaking, the relevant equilibrium concept is Perfect Bayesian Equilibrium, but we ommit specifying beliefs as they are trivial: the manager's beliefs are the same as the prior, and the candidate observes the standard and her type.

expected loss from rejection is strictly less than the expected loss from hiring. In contrast, the standard set by the manager in the Bayesian Nash equilibrium (BNE) of the static game Γ_0, in which the candidate does not observe the standard before choosing her readiness, is ex-post efficient. The Appendix provides a brief analysis of the static game and its BNE, denoted by $(s^*_{NE}, \theta^*_{uNE}, \theta^*_{qNE})$. Our focus, however, is on the dynamic game Γ_1.

For any SPNE $(s^*_P, \theta^*_u, \theta^*_q)$ with $s^*_P \in (0,1)$, if F is submodular at $(s^*_P, \theta^*_u(s^*_P))$ and supermodular at $(s^*_P, \theta^*_q(s^*_P))$, then the manager is soft:[23] if $s^*_P \geq s^*(\theta^*_u(s^*_P), \theta^*_q(s^*_P); \kappa)$, then the manager could increase his expected payoff by a marginal decrease in the standard, as this would reduce information undermining and increase information generation, and improve (or keep nearly unchanged) ex-post efficiency, contradicting that s^*_P is an equilibrium standard. Analogously, the manager is harsh in equilibria such that $(s^*_P, \theta^*_u(s^*_P))$ and $(s^*_P, \theta^*_q(s^*_P))$ are located in the supermodular and submodular regions of F, respectively.[24]

If both $(s^*_P, \theta^*_u(s^*_P))$ and $(s^*_P, \theta^*_q(s^*_P))$ are located in the submodular region of F, or both are located in the supermodular region of F, then the relative magnitudes of the strategic effects of the standard on each type of candidate's effort play a critical role. In particular, we define the *strategic ratio*

$$r(s) := F_\theta(s, \theta^*_q(s)) \frac{d\theta^*_q(s)}{ds} \left(F_\theta(s, \theta^*_u(s)) \frac{d\theta^*_u(s)}{ds} \right)^{-1}$$

for all $s \in (0,1) \setminus \{\hat{s}_u\}$, and $r(0) := \lim_{s \to 0} r(s)$, $r(\hat{s}_u) := \lim_{s \to \hat{s}_u} r(s)$ and $r(1) := \lim_{s \to 1} r(s)$, respectively, whenever these limits exist. Thus, the strategic ratio is the ratio of the strategic effect of the standard on the signal distribution of the fit candidate, divided by the corresponding effect on the signal distribution of the unfit candidate.

Lemma 2 *Assume F.1-F.2 and C.1. For any SPNE $(s^*_P, \theta^*_u, \theta^*_q)$ of Γ_1 with $s^*_P \in (0,1)$:*

(i) if $(s^*_P, \theta^*_u(s^*_P))$ and $(s^*_P, \theta^*_q(s^*_P))$ are located in the submodular and supermodular regions of F, respectively, i.e., $s^*_P \in (\hat{s}_q, \hat{s}_u)$, then the manager is soft;

(ii) if $(s^*_P, \theta^*_u(s^*_P))$ and $(s^*_P, \theta^*_q(s^*_P))$ are located in the supermodular and submodular regions of F, respectively, i.e., $s^*_P \in (\hat{s}_u, \hat{s}_q)$, then the manager is harsh;

(iii) if both $(s^*_P, \theta^*_u(s^*_P))$ and $(s^*_P, \theta^*_q(s^*_P))$ are located in the submodular region,

[23] For instance, if $\widetilde{s}$ is strictly decreasing, then this is the case for $s^*_P \in (\hat{s}_q, \hat{s}_u)$.

[24] If $\widetilde{s}$ is strictly increasing, then this is the case for $s^*_P \in (\hat{s}_u, \hat{s}_q)$.

i.e., $s_P^* \in (0, \min\{\hat{s}_q, \hat{s}_u\})$*, then the manager is soft (harsh) if and only if* $r(s_P^*) < (>)\kappa$*; and*

(iv) if both $(s_P^*, \theta_u^*(s_P^*))$ *and* $(s_P^*, \theta_q^*(s_P^*))$ *are located in the supermodular region, i.e.,* $s_P^* \in (\max\{\hat{s}_q, \hat{s}_u\}, 1)$*, then the manager is soft (harsh) if and only if* $r(s_P^*) > (<)\kappa$*.*

The proof of Lemma 2 follows directly from Remark 1 and (7). Now we discuss the intuition of parts (iii) and (iv) for the case in which $r(s_P^*) < \kappa$ (the intuition of the case $r(s_P^*) > \kappa$ is analogous). If the strategic ratio is less than the prior, then the manager is relatively more concerned with the effect of his commitment on the effort exerted by the unfit candidate. Since the effort exerted by the unfit candidate is a strategic complement of the standard over the submodular region and a strategic substitute over the supermodular region, the manager optimally commits to soft and harsh standards, respectively, in those regions.

In the sequel, we let $\kappa \rightrightarrows \mathcal{S}^*(\kappa)$ be the correspondence mapping $\kappa \in (0, \infty)$ to the set of standards in a pure strategy SPNE, and $\mathcal{S}^{*-1}$ its inverse.[25] We say that $\mathcal{S}^*$ is *weakly* (*strictly*) *increasing* within a given interval $I \subseteq (0, \infty)$ if $\kappa' > \kappa$, $s \in \mathcal{S}^*(\kappa)$ and $s' \in \mathcal{S}^*(\kappa')$ imply that $s' \geq s$ ($s' > s$), for all $\kappa, \kappa' \in I$. These properties will play a role in our analysis.

3.3 Confirmativism and Conservatism

Below we study whether deviations from ex-post optimality of the standard under commitment bias the manager towards or against the alternative he would choose according to his prior beliefs only. The definitions of confirmativism and conservatism (c.f., Li (2001)) are useful in our analysis:

Definition 2 *Let* $(s_P^*, \theta_u^*, \theta_q^*)$ *be an equilibrium of* Γ_1*. The manager is* confirmative *at* $(s_P^*, \theta_u^*, \theta_q^*)$ *if* $\kappa > (<)1$ *and he is harsh (soft). The manager is* conservative *at* $(s_P^*, \theta_u^*, \theta_q^*)$ *if* $\kappa > (<)1$ *and he is soft (harsh).*

We also say that the manager is *confirmative at* κ if he is confirmative at $(s, \theta_u^*, \theta_q^*)$ for all $s \in \mathcal{S}^*(\kappa)$. *Conservative* and *ex-post efficient at* κ are defined analogously. Finally, we say that the manager of a game Γ_1 is *uniformly confirmative* if he is confirmative for some priors and he is not conservative for any prior. A *uniformly conservative* manager is defined in the same manner.

[25] We typically have a unique equilibrium in Γ_1. Some of these games, however, have multiple equilibria for some knife-edge values of the prior κ.

Our results in the following sections reveal that optimal standards tend to be confirmative when information undermining is stronger than information generation and conservative in the opposite case.

4 Economics of Information Undermining

It is instructive to start considering the case with no information generation.

4.1 Pure Information Undermining

A game in which the fit candidate's readiness is exogenously given, is called a *pure information undermining game*. In this game, denoted by Γ_2, $\Theta_q = \{\underline{\theta}_q\}$ for some $\underline{\theta}_q \in \left(\theta_u^*(\hat{s}_u), \overline{\theta}\right]$, and the unfit candidate exerts effort to make her signal distribution more similar to the fit candidate's signal distribution. In pure information undermining games, just as in the solution of the classical statistical problem, the equilibrium standard is increasing in the prior.

Lemma 3 *Assume that F satisfies F.1-F.2 and C_u satisfies C.1 (i)-(iv). In every game Γ_2, $\mathcal{S}^*$ is weakly increasing over $(0,\infty)$ and strictly increasing over $\mathcal{S}^{*-1}(0,1)$. Further, $\mathcal{S}^*(\kappa) = \{0\}$ if $\kappa \in (0, \underline{\kappa}]$ and $\mathcal{S}^*(\kappa) = \{1\}$ if $\kappa \in [\overline{\kappa}, \infty)$.*

Consider a SPNE standard, $s_P^* \in (0,1)$. From (5) and (6), v and $g(\cdot, \theta_u^*(\cdot), \underline{\theta}_q)$ cross only once, and $s_P^* < \hat{s}_u$ if and only if $v(s_P^*) > g(s_P^*, \theta_u^*(s_P^*), \underline{\theta}_q)$:[26] since $g(\cdot, \theta_u^*(s_P^*), \underline{\theta}_q)$ is increasing, the ex-post optimal standard is higher than the equilibrium standard. Similarly, $s_P^* > \hat{s}_u$ if and only if $v(s_P^*) < g(s_P^*, \theta_u^*(s_P^*), \underline{\theta}_q)$. The left and right panel of Figure 2 show v and $g(\cdot, \theta_u^*(\cdot), \underline{\theta}_q)$ from Example 2 (provided below), for different values of $\underline{\theta}_q$.

Proposition 1 *Assume that F satisfies F.1-F.2 and C_u satisfies C.1 (i)-(iv). For any game Γ_2, there exists $\widetilde{\kappa}_U \in (\underline{\kappa}, \overline{\kappa})$ such that*

$$\text{the manager is} \begin{cases} \textit{ex-post efficient} & \text{if } \kappa \in (0, \underline{\kappa}] \\ \textit{soft} & \text{if } \kappa \in (\underline{\kappa}, \widetilde{\kappa}_U) \\ \textit{harsh} & \text{if } \kappa \in (\widetilde{\kappa}_U, \overline{\kappa}) \\ \textit{ex-post efficient} & \text{if } \kappa \in [\overline{\kappa}, \infty). \end{cases} \tag{8}$$

[26] If $\frac{dF(s,\theta_u^*(s))}{ds} \leq 0$ for some $s \in (0,1)$, then from (7), we have $\frac{dV(s,\theta_u^*(s),\underline{\theta}_q)}{ds} > 0$, and, hence, s cannot be an equilibrium standard. All examples in the paper satisfy that $v(s) > 0$ for all $s \in (0,1)$. There are, however, games such that $v(s) < 0$ within some intervals. For instance, in the game Γ_2, defined by $F(s,\theta) = \theta s^{10} + (1-\theta)s$, $\underline{\theta}_q = 1$, and $C_u(\theta) = \frac{1}{2}\theta^2$ for all $s \in [0,1]$ and $\theta \in \Theta = [0,1]$, we have that $v(s) < 0$ for all $s \in (0.52, 0.65)$.

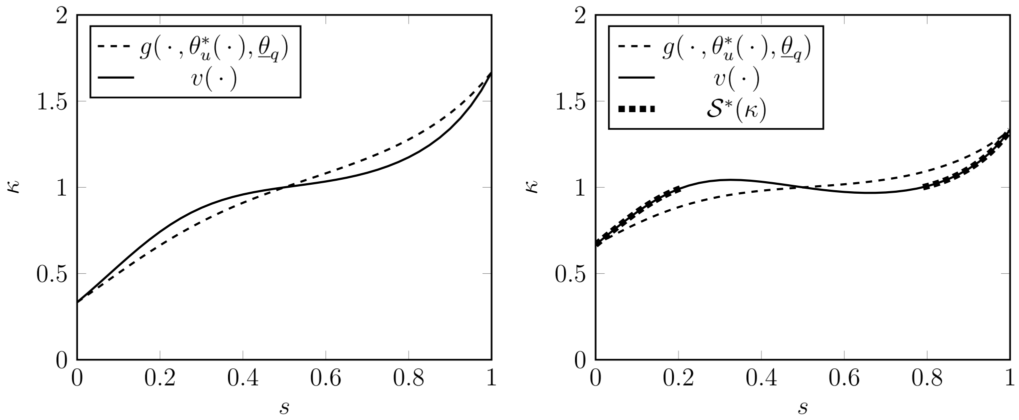

Figure 2: **Example 2.** Left and right panel: v (solid line) and equilibrium standards for each κ in the static game Γ_0 (dashed line) for $\underline{\theta}_q = \frac{2}{3}$ (left panel) and $\underline{\theta}_q = \frac{1}{3}$ (right panel). In the left panel, the equilibrium standards of Γ_2 coincide with v and in the right panel they correspond to the dashdotted line.

The proof of Proposition 1 builds upon Lemmata 1-3: by Lemma 3, relatively low priors lead to relatively low standards in equilibrium. By Lemma 1, if the equilibrium standard is relatively low, then the equilibrium standard and unfit readiness pair $(s_P^*, \theta_u^*(s_P^*))$ is located in the submodular region of F, and, hence, by part (i) of Lemma 2, the manager is soft.[27] An analogous argument reveals that relatively high priors lead to a harsh standard.

The manager is ex-post efficient if the evidence from the test is not valuable because it cannot overturn prior beliefs; i.e., if $\kappa \in (0, \underline{\kappa}] \cup [\overline{\kappa}, \infty)$; or, if the evidence is valuable but a commitment to a standard is not; i.e., if $\kappa = \widetilde{\kappa}_U$ and $\mathcal{S}^*(\widetilde{\kappa}_U) = \{\hat{s}_u\}$.[28]

As the unfit candidate's cost of improving readiness increases, she becomes less responsive to changes in the standard. In the Appendix we provide sufficient conditions for relatively high costs C_u to generate a monotone pseudo likelihood ratio function v (see Remark 4).[29] If v is strictly increasing, then $\mathcal{S}^*$ is single-valued at all κ, and the cut-off prior for soft and harsh standards is equal to the likelihood ratio function evaluated at the modularity-switch point, $\hat{\kappa} := g(\hat{s}_u, \theta_u^*(\hat{s}_u), \underline{\theta}_q)$, as illustrated in the left-panel of Figure 2. In contrast, if v is strictly decreasing over

[27]Since for game Γ_2 we have $\frac{d\theta_q^*(s)}{ds} = 0$, the argument leading to Lemma 2, in this game, yields that, if $(s_P^*, \theta_u^*, \underline{\theta}_q)$ is a SPNE of Γ_2 with $s_P^* \in (0,1)$, then the equilibrium standard is soft (harsh) if and only if F is submodular (supermodular) at $(s_P^*, \theta_u^*(s_P^*))$.

[28]Example 2 below illustrates that the manager may not be ex-post efficient at $\widetilde{\kappa}_U$ when $\mathcal{S}^*(\widetilde{\kappa}_U)$ is multivalued.

[29]In contrast, $g(\cdot, \theta_u^*(\cdot), \underline{\theta}_q)$ is strictly increasing, as (5) and direct computations reveal.

some intervals, the equilibrium standard varies discontinuously with changes in κ.[30] The following remark summarizes these observations.

Remark 2 *Assume that F satisfies F.1-F.2 and C_u satisfies C.1 (i)-(iv). Consider any game Γ_2.*

(i) If $v(s) < (>)\hat{\kappa}$ for all $s \in (0, \hat{s}_u)$ $(s \in (\hat{s}_u, 1))$, then $\widetilde{\kappa}_U = \hat{\kappa}$. Therefore, Proposition 1 (and Corollary 1 below) hold, mutatis mutandis, replacing $\widetilde{\kappa}_U$ by $\hat{\kappa}$. Further, $\mathcal{S}^*(\hat{\kappa}) = \{\hat{s}_u\}$ and thus, the manager is ex-post efficient at $\hat{\kappa}$.

(ii) Suppose that v is strictly decreasing over some interval $(\underline{s}, \overline{s})$, with $0 < \underline{s} < \overline{s} < 1$. Then, there exists a prior $\kappa \in (0, \infty)$ and $\delta > 0$ such that $\kappa' < \kappa < \kappa''$, $s' \in \mathcal{S}^*(\kappa')$, and $s'' \in \mathcal{S}^*(\kappa'')$ imply that $s'' - s' > \delta$.

A sufficient condition for part (i) is that v is strictly increasing. Part (ii) implies that if v is non-monotone, candidates with arbitrarily similar priors may be subject to very different standards. Our next example illustrates Proposition 1 and Remark 2.

Example 2 *Consider $F(z, \theta) = \theta z^2 + (1-\theta)z$ for all $z \in [0, 1]$ and $\theta \in \Theta = [0, 1]$. Assume $\underline{\theta}_q \in \left(\frac{1}{4}, 1\right]$. Thus, $\underline{\kappa} = 1 - \underline{\theta}_q$ and $\overline{\kappa} = 1 + \underline{\theta}_q$. Since $\widetilde{s}(\theta) = \frac{1}{2}$ for all $\theta < \overline{\theta}$, the modularity-switch point is $\hat{s}_u = \frac{1}{2}$. The unfit candidate's cost function is $C_u(\theta) = \frac{1}{2}\theta^2$ for all $\theta \in \Theta$. Hence, $\theta_u^*(s) = s(1-s)$ for all $s \in [0, 1]$. By Proposition 1, in the game Γ_2, the manager is soft for $\kappa \in (1 - \underline{\theta}_q, \widetilde{\kappa}_U)$, harsh for $\kappa \in (\widetilde{\kappa}_U, 1 + \underline{\theta}_q)$, and ex-post efficient for $\kappa \leq 1 - \underline{\theta}_q$ and $\kappa \geq 1 + \underline{\theta}_q$.*

For $\underline{\theta}_q \in \left(\frac{1}{2}, 1\right]$, direct computations reveal that $v'(s) > 0$ for all $s \in (0, 1)$. Therefore, by Remark 2, $\widetilde{\kappa}_U = g(\hat{s}_u, \theta_u^*(\hat{s}_u), \underline{\theta}_q) = 1$ and the manager is ex-post efficient at $\widetilde{\kappa}_U$. For $\underline{\theta}_q \in \left(\frac{1}{4}, \frac{1}{2}\right)$, v is non-monotone. In this case, $\widetilde{\kappa}_U = 1$ and $\mathcal{S}^*(\widetilde{\kappa}_U) = \{s_1, s_2\}$, where s_1 and s_2 are, respectively, the smallest and greatest root of $s(1-s) = \frac{\underline{\theta}_q}{2}$. The right-panel of Figure 2 shows how the standard varies discontinuously with κ for $\underline{\theta}_q = \frac{1}{3}$, with $s_1 = 0.21$ and $s_2 = 0.79$. The manager is not ex-post efficient at $\widetilde{\kappa}_U$ —he is soft at $(0.21, \theta_u^*, \theta_q^*)$ and harsh at $(0.79, \theta_u^*, \theta_q^*)$.

4.2 Confirmativism and Conservatism under Information Undermining

Commitment to a standard can involve a mix of confirmativism and conservatism when the manager faces candidates with different priors. Proposition 1 and Definition 2 imply:

[30] At critical points of $V(\cdot, \theta_u^*(\cdot), \underline{\theta}_q)$ located in intervals where v is decreasing, the manager's expected loss has a local maximum.

Corollary 1 *Assume that F satisfies F.1-F.2 and C_u satisfies C.1 (i)-(iv). For every game Γ_2 and $\kappa \neq \widetilde{\kappa}_U$, the manager is (i) confirmative at κ if and only if $\underline{\kappa} < \kappa < \min\{1, \widetilde{\kappa}_U\}$ or $\max\{1, \widetilde{\kappa}_U\} < \kappa < \overline{\kappa}$; and (ii) conservative at κ if and only if $\min\{1, \widetilde{\kappa}_U\} < \kappa < \max\{1, \widetilde{\kappa}_U\}$.*

A direct consequence of Corollary 1 is that, under assumptions F.1-F.2 and C.1, in any game Γ_2, (i) the manager is uniformly confirmative if and only if $\widetilde{\kappa}_U = 1$ and (ii) the manager cannot be uniformly conservative.

If $\frac{d\widetilde{s}}{d\theta} > (<)0$, as in Example 1, then $\hat{\kappa} < (>)1$;[31] thus, if we also have that v is strictly increasing, then the manager uses a mix of confirmative and conservative standards (depending on his prior beliefs). The following example illustrates this.

Example 3 *(Example 1 revisited). Assume that $\underline{\theta}_q \in (1, 2]$. Recall that $g(z, \theta, \underline{\theta}_q) = \frac{\underline{\theta}_q}{\theta} z^{\underline{\theta}_q - \theta}$ for all $z \in (0, 1)$ and $\theta \in \Theta$, and that $(\hat{s}_u, \theta_u^*(\hat{s}_u)) = (\mathrm{e}^{-1}, 1)$. Thus, $\hat{\kappa} \equiv g(\hat{s}_u, \theta_u^*(\hat{s}_u), \underline{\theta}_q) = \underline{\theta}_q \mathrm{e}^{1-\underline{\theta}_q} < 1$.*

For $\underline{\theta}_q \in [1.28, 2]$, routine computations reveal that $v' > 0$ over $(0, 1)$. Thus, part (i) of Remark 2 implies that $\widetilde{\kappa}_U = \hat{\kappa}$. Therefore, the manager is confirmative if and only if $\kappa \in (0, \hat{\kappa}) \cup \left(1, \frac{\underline{\theta}_q}{\underline{\theta}}\right)$ and conservative if and only if $\kappa \in (\hat{\kappa}, 1)$, by Corollary 1 and Remark 2. The manager is ex-post efficient if and only if $\kappa = \hat{\kappa}$ or $\kappa \in \left[\frac{\underline{\theta}_q}{\underline{\theta}}, \infty\right)$. The ranges of κ for which the manager is confirmative and conservative for $\underline{\theta}_q = 2$ are illustrated in Figure 3.

If F has a neutral signal,[32] $s^\star$ (c.f., Milgrom (1981)), then $\hat{\kappa} = 1$. Thus, if v is strictly increasing, then the manager does not use conservative standards. In general, however, having a neutral signal is not sufficient for F to generate uniform confirmativism for all cost function.[33] A stronger condition that is sufficient is quasi-symmetry: a cumulative distribution F is *quasi-symmetric* (QS) if $F_\theta(s, \theta) = F_\theta(s', \theta)$ for some $\theta \in \Theta^\circ$ implies that $F_\theta(s, \theta') = F_\theta(s', \theta')$ for all $\theta' \in \Theta^\circ$, for all $s, s' \in [0, 1]$. If F is QS, then it has a neutral signal $s^\star$, and for each standard $s < (>)s^\star$, there is a standard $s_f(s) > (<)s^\star$ such that, for all cost function C_u satisfying Assumption C.1, we have: $\theta_u^*(s) = \theta_u^*(s_f(s))$ and $V(s, \theta_u^*(s), \underline{\theta}_q; 1) =$

[31] For all $\theta < \underline{\theta}_q$, we have that $\frac{dg(\widetilde{s}(\theta), \theta, \underline{\theta}_q)}{d\theta} = \frac{\partial g(\widetilde{s}(\theta), \theta, \underline{\theta}_q)}{\partial s} \frac{d\widetilde{s}(\theta)}{d\theta}$. Since $\lim_{\theta \to \underline{\theta}_q} g(\widetilde{s}(\theta), \theta, \underline{\theta}_q) = 1$, $\frac{dg(\widetilde{s}(\theta), \theta, \underline{\theta}_q)}{d\theta} > (<)0$ for all $\theta < \underline{\theta}_q$ implies that $g(\widetilde{s}(\theta), \theta, \underline{\theta}_q) < (>)1$ for all $\theta < \underline{\theta}_q$, which, in turn, yields $\hat{\kappa} < (>)1$.

[32] We say that a distribution F *has a neutral signal* $s^\star \in (0, 1)$ if $f(s^\star, \theta) = f(s^\star, \theta')$ for every θ, $\theta' \in \Theta$. The prior and posterior beliefs associated to a neutral signal are the same, regardless of the readiness of candidates. If $s^\star$ is the neutral signal of F, then the modularity-switch point of any game Γ_2 with distribution F is the neutral signal, $\hat{s}_u = s^\star$.

[33] It is not difficult to construct examples of distributions having a neutral signal, which nevertheless, lead to conservatism for some cost functions and priors.

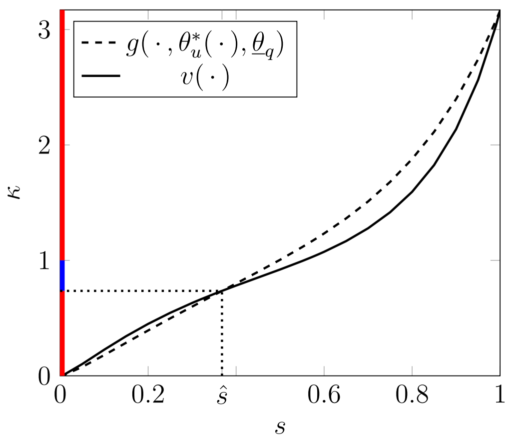

Figure 3: **Example 3.** Equilibrium standards in Γ_2 (solid line) and Γ_0 (dashed line) for $\underline{\theta}_q = 2$. In Γ_2, the manager is conservative (confirmative) at the priors highlighted in blue (red).

$V(s_f(s), \theta_u^*(s_f(s)), \underline{\theta}_q; 1)$ for all $s \in [0,1]$ and $\underline{\theta}_q \in (\theta_u^*(\hat{s}_u), \overline{\theta}]$ (for details, refer to Claim 2 in the Appendix).

In economic terms, for QS distributions, both the structure of incentives of the manager and the unfit candidate exhibit symmetries around the neutral signal —the signal that conveys neither good nor bad news. These symmetries imply $\mathcal{S}^*(1) = \{s^\star\}$ or $s, s_f(s) \in \mathcal{S}^*(1)$ for some $s < s^\star$; and hence, by Lemma 3, $\widetilde{\kappa}_U = 1$. Thus, we have proved the following:

Proposition 2 *Assume F.1-F.2. If F is QS, then the manager of a game Γ_2 is uniformly confirmative for all cost function C_u satisfying Assumption C.1 (i)-(iv).*

The policy insight of Proposition 2 is that, provided that the incentives symmetries around the neutral signal described above hold for F, then, regardless of the cost structure of information undermining, at the margin, the decision favoured by prior beliefs should be upheld. This partial disregard of the information contained in the evidence desincentivizes effort by the low type agent. This holds whether the prior favors the action preferred by the agent —in which case confirmativism dictates to hire a fraction of candidates that otherwise would be rejected— or whether the prior opposes to this action —in which case confirmativism dictates to reject a fraction of candidates that otherwise would be hired.

In the Appendix we provide three simple properties on the functional form of F that are sufficient for QS. Intuitively, F is QS whenever the *candidate's marginal return to readiness*, $-F_\theta(z,\theta)$, is (i) independent of θ, for all $z \in [0,1]$, (ii) multiplicatively separable, or (iii) plainly, symmetric around the neutral signal.

For instance, consider the *rotation distribution* (RD) family, defined by $f(z,\theta) = \gamma(\theta)f_1(z) + (1-\gamma(\theta))f_0(z)$ for all $(z,\theta) \in [0,1]^2$, where $\gamma : \Theta \to [0,1]$ is strictly increasing, and the densities f_0 and f_1 satisfy $d(f_1/f_0)/dz > 0$. Any F in the RD family (as in Example 2) is QS because it satisfies the second property.[34] Therefore, the manager is uniformly confirmative by Proposition 2.

4.3 Dominating Information Undermining

Most qualitative aspects of the results that we derived for game Γ_2 arise in game Γ_1 when the manager's strategic concerns are dominated by the effect of the standard on the effort exerted by the unfit candidate. For instance, if returns to readiness are decreasing and the fit candidate has a high natural readiness, she may benefit little from exerting effort: if the marginal benefit from effort vanishes as $\underline{\theta}_q \to \bar{\theta}$ (i.e., if $F_\theta(\cdot,\overline{\theta}) = 0$), then the strategic ratio goes to 0 and the manager becomes soft for relatively low priors and harsh for relatively high priors (by Lemmata 1-2). Then, the thesis of Proposition 1 for the pure information undermining model remains valid in the general model. Additionally, if F is QS, the manager is uniformly confirmative.

Our formal proof relies on the differentiability of v, which requires the cost functions to be thrice differentiable, and slightly strengthening Assumption F.1, ensuring that the derivatives of F, up to the third order, are bounded.

Assumption C. 2 *C_i'' is continuously differentiable and $C_i'' > 0$ for $i = u, q$.*

Assumption F.3 *The third-order partial derivatives of F are continuous real functions defined over D, and $F(\cdot,\theta)$ is atomless with $f > 0$ for all $\theta \in \Theta$.*

Let $\Gamma_1(\underline{\theta}_q)$ be the game Γ_1 defined by a triplet (F, C_q, C_u), with $\underline{\theta}_q \in \Theta^\circ$. In the sequel, when convenient, we write explicitly the dependence of $\underline{\kappa}$ and $\overline{\kappa}$ on $\underline{\theta}_q$, writing $\underline{\kappa}(\underline{\theta}_q)$ and $\overline{\kappa}(\underline{\theta}_q)$, respectively.

Proposition 3 *Assume F.2-F.3, C.1-C.2, and $F_\theta(s,\overline{\theta}) = 0$ for all $s \in (0,1)$. Then, there exists $\underline{\underline{\theta}}_q \in \Theta^\circ$ such that for all $\underline{\theta}_q \in \left(\underline{\underline{\theta}}_q, \overline{\theta}\right)$, (i) there exists $\widetilde{\kappa}(\underline{\theta}_q) \in (\underline{\kappa}(\underline{\theta}_q), \overline{\kappa}(\underline{\theta}_q))$ such that (8) holds in the game $\Gamma_1(\underline{\theta}_q)$, replacing $\widetilde{\kappa}_U$ with $\widetilde{\kappa}(\underline{\theta}_q)$; and (ii) if F is QS, then the manager is uniformly confirmative.*

The proof shows how the arguments in the proofs of Propositions 1 and 2 for the game Γ_2, can be extended to Γ_1, for large $\underline{\theta}_q$: as $\underline{\theta}_q \to \overline{\theta}$, v approaches to $\frac{f(\cdot,\overline{\theta})}{d_u(\cdot)}$

[34] A distribution in the RD family also satisfies the first property if γ is an affine transformation of θ, as, for instance, in Example 2.

and $g(\cdot, \theta_u^*(\cdot), \theta_q^*(\cdot))$ approaches to $g(\cdot, \theta_u^*(\cdot), \overline{\theta})$; thus, v and $g(\cdot, \theta_u^*(\cdot), \theta_q^*(\cdot))$ cross only once over $(0, 1)$ —just as in the analysis of game Γ_2.

5 Economics of Information Generation

In this section, we consider the case in which the manager's strategic concerns are dominated by the effect of the standard on the fit candidate's effort.[35]

5.1 Pure Information Generation

It is instructive to first analyze a *pure information generation game*, denoted by Γ_3, in which the unfit candidate is non-strategic: her readiness is exogenously given by her natural readiness; i.e., $\theta_u^*(s) = \underline{\theta}$ for all $s \in [0, 1]$. All other assumptions on Γ_1, laid out in Section 2, remain valid. Loosely speaking, Γ_3 is the game obtained from any game Γ_1, as $C_u(\theta)$ goes to infinity for all $\theta > \underline{\theta}$.

Let $\underline{\kappa}_C := \inf_{s\in(0,1)} \left\{ \frac{F(s,\theta_q^*(s))}{F(s,\underline{\theta})} \right\}$, $\overline{\kappa}_C := \sup_{s\in(0,1)} \left\{ \frac{1-F(s,\theta_q^*(s))}{1-F(s,\underline{\theta})} \right\}$, and $\widetilde{\kappa}_G := g(\hat{s}_q, \underline{\theta}, \theta_q^*(\hat{s}_q))$. In contrast to the pure information undermining scenario (Proposition 1), here optimal standards are harsh for relatively low priors and soft for relatively high priors.

Proposition 4 *Assume that F satisfies F.1-F.2 and C_q satisfies C.1 (i)-(iv). For any game Γ_3, $\underline{\kappa}_C < \widetilde{\kappa}_G < \overline{\kappa}_C$ and*

$$\text{the manager is} \begin{cases} \textit{ex-post efficient} & \textit{if } \kappa \in (0, \underline{\kappa}_C) \\ \textit{harsh} & \textit{if } \kappa \in (\underline{\kappa}_C, \widetilde{\kappa}_G) \\ \textit{ex-post efficient} & \textit{if } \kappa = \widetilde{\kappa}_G \\ \textit{soft} & \textit{if } \kappa \in (\widetilde{\kappa}_G, \overline{\kappa}_C) \\ \textit{ex-post efficient} & \textit{if } \kappa \in (\overline{\kappa}_C, \infty). \end{cases} \tag{9}$$

The proof builds on Lemmata 1, 2, and 7 (a lemma fairly analogous to Lemma 3 that is provided in the Appendix),[36] in the same manner as the proof of Propo-

[35] One example is in-program selection in graduate school programs of subjects requiring specific skills or background (e.g., economics, mathematics, and engineering). Many schools use qualifying or preliminary exams. Qualified candidates' readiness for the exams can increase substantially with their exerted effort, which is likely to be determined by approval cut-offs. In contrast, unqualified candidates' readiness may increase very little due to lack of skills or a weak background (for an empirical analysis of the determinants of success in qualifying exams, thesis completion, and research productivity in economics Ph.D. programs, see Grove and Wu (2007)).

[36] Since for game Γ_3 we have $\frac{d\theta_u^*(s)}{ds} = 0$, the argument leading to Lemma 2, in this game, yields that, if $(s_P^*, \underline{\theta}, \theta_q^*)$ is a SPNE of Γ_3 with $s_P^* \in (0, 1)$, then the equilibrium standard is harsh (soft) if and only if F is submodular (supermodular) at $(s_P^*, \theta_q^*(s_P^*))$.

sition 1 builds on Lemmata 1-3. As this proof reveals, $\underline{\kappa}_C \leq \underline{\kappa}$ and $\overline{\kappa} \leq \overline{\kappa}_C$: information generation enlarges the set of priors for which testing is worthy.

In the pure information generation game, as in game Γ_2, commitment can involve a mix of confirmativism and conservatism when the manager faces candidates with different priors. The deviations, however, are in the opposite direction.

Corollary 2 *Assume that F satisfies F.1-F.2 and C_q satisfies C.1 (i)-(iv). For all game Γ_3 and $\kappa \notin \{\underline{\kappa}_C, \overline{\kappa}_C\}$ the manager is (i) conservative at κ if and only if $\underline{\kappa}_C < \kappa < \min\{1, \widetilde{\kappa}_G\}$ or $\max\{1, \widetilde{\kappa}_G\} < \kappa < \overline{\kappa}_C$, and (ii) confirmative at κ if and only if $\min\{1, \widetilde{\kappa}_G\} < \kappa < \max\{1, \widetilde{\kappa}_G\}$.*

A direct consequence of Corollary 2 is that under Assumptions F.1-F.2 and C.1 (i)-(iv), for any Γ_3, (i) the manager is uniformly conservative if and only if $\widetilde{\kappa}_G = 1$ and (ii) the manager cannot be uniformly confirmative.

In contrast to the pure information undermining setting, in the pure information generation model, the optimal standard associated to the cut-off prior is always the modularity-switch point; that is, $\mathcal{S}^*(\widetilde{\kappa}_G) = \{\hat{s}_q\}$ (see the proof of Proposition 4). This feature of the problem allows us to provide a sufficient condition for uniform conservatism: if F has a neutral signal, then $\hat{s}_q = s^\star$ and $\widetilde{\kappa}_G = 1$.

Corollary 3 *Assume that F has a neutral signal and satisfies F.1-F.2. The manager is uniformly conservative in game Γ_3, for all natural readiness $\underline{\theta}_q \in \Theta^\circ$ and cost function C_q satisfying Assumption C.1 (i)-(iv).*

5.2 Dominating Information Generation

The main qualitative features of game Γ_3 arise in game Γ_1, provided that the unfit candidate's effort is sufficiently costly. For any game Γ_1, defined by a triplet $(F, C_q(\cdot; \underline{\theta}_q), C_u)$, we define a set of games indexed by $\lambda \in [0, 1]$, and denoted by $\Gamma_1(\lambda)$, where the only difference between Γ_1 and $\Gamma_1(\lambda)$ is that the cost function of the unfit candidate in the latter is given by $\lambda^{-1}C_u$, for all $\lambda \in (0, 1]$, whereas $\Gamma_1(0)$ corresponds to Γ_3. Let $\underline{\kappa}_C(\lambda) := \inf_{s\in(0,1)} \left\{ \frac{F(s,\theta_q^*(s))}{F(s,\theta_u^*(s;\lambda))} \right\}$ and $\overline{\kappa}_C(\lambda) := \sup_{s\in(0,1)} \left\{ \frac{1-F(s,\theta_q^*(s))}{1-F(s,\theta_u^*(s;\lambda))} \right\}$, where $\theta_u^*(\cdot; \lambda)$ is the best response of the unfit candidate with cost function $\lambda^{-1}C_u$ for all $\lambda \in (0, 1]$, and $\theta_u^*(\cdot; 0) = \underline{\theta}$.

Proposition 5 *Assume F.2-F.3 and C.1-C.2. Then, there exists $\overline{\lambda} > 0$ such that for all $\lambda \in (0, \overline{\lambda})$, (i) there exists $\widetilde{\kappa}_G(\lambda) \in (\underline{\kappa}_C(\lambda), \overline{\kappa}_C(\lambda))$ such that (9) holds in the game $\Gamma_1(\lambda)$, mutatis mutandis, replacing $\widetilde{\kappa}_G$, $\underline{\kappa}_C$, and $\overline{\kappa}_C$, with $\widetilde{\kappa}_G(\lambda)$, $\underline{\kappa}_C(\lambda)$, and $\overline{\kappa}_C(\lambda)$, respectively; and (ii) if F has a neutral signal, then the manager is uniformly conservative.*

5.3 Undominated Effects and Multiple Cut-offs

For intermediate values of $\underline{\theta}_q$ and λ, multiple soft-harsh or harsh-soft cut-offs may arise —contrasting with the single cut-offs of dominating information undermining and generation games. For instance, consider (i) F in the RD family, with $\gamma(\theta) = \theta\left(1 - \frac{\theta}{2}\right)$ for all $\theta \in \Theta = [0, 1]$, and $f_0(z) = 1$ and $f_1(z) = 2z$ for all $z \in [0, 1]$; and (ii) $C_u(\theta) = \frac{1}{2}\theta^2$ for all $\theta \in [0, 1]$ and $C_q(\theta) = \frac{1}{2}(\theta - \underline{\theta}_q)^2$ for all $\theta \in [\underline{\theta}_q, 1]$ and $\underline{\theta}_q \in (0, 1)$. In the game $\Gamma_1(\lambda)$ with $\lambda = 0.5$, for $\underline{\theta}_q \in (0.16, 0.23)$ a harsh-soft-harsh-soft pattern arises as κ increases. Similarly, for $\underline{\theta}_q \in (0.23, 0.42)$ we have a harsh-soft-harsh pattern. Multiple cut-offs rule out uniform confirmativism or uniform conservatism.

6 Welfare Analysis

Now we analyze the impact on welfare of the manager's ability to commit. We focus on Pareto dominance results, which we find for some configurations of dominating information undermining and generation with suitable prior beliefs.

We compare equilibrium payoffs in the dynamic game Γ_1 and the static game Γ_0. The manager is weakly better-off when he can commit. From the envelope theorem, candidates are strictly better-off in an equilibrium of game Γ_1 than in an equilibrium of Γ_0 —and hence Γ_1 Pareto dominates Γ_0— if and only if the standard in the former is lower than in the latter.

We present two welfare results on the effect of commitment; the first deals with the case in which the effect of standards on the unfit candidate's effort dominates.

Corollary 4 *Assume F.2-F.3, C.1-C.2, and* $F_\theta(z, \overline{\theta}) = 0$ *for all* $z \in (0, 1)$*. Then, there exists* $\underline{\underline{\theta}}_q \in \Theta^\circ$ *satisfying that, for all* $\underline{\theta}_q \in \left(\underline{\underline{\theta}}_q, \overline{\theta}\right]$*, there exists* $\widetilde{\kappa}(\underline{\theta}_q) \in (\underline{\kappa}(\underline{\theta}_q), \overline{\kappa}(\underline{\theta}_q))$ *such that, for all* $\kappa \in (\underline{\kappa}(\underline{\theta}_q), \widetilde{\kappa}(\underline{\theta}_q))$ *and SPNE* $(s_P^*, \theta_u^*, \theta_q^*(\cdot; \underline{\theta}_q))$ *of* $\Gamma_1(\underline{\theta}_q)$*, there is a BNE of* $\Gamma_0(\underline{\theta}_q)$ *that is Pareto dominated by* $(s_P^*, \theta_u^*, \theta_q^*(\cdot; \underline{\theta}_q))$*. Furthermore, if (1) also holds at* $(0, \theta)$ *and* $(1, \theta)$ *for all* $\theta \in \Theta^\circ$*, then* $\underline{\underline{\theta}}_q \in \Theta^\circ$ *can be chosen so that the BNE of* $\Gamma_0(\underline{\theta}_q)$ *is unique.*[37]

The first part follows from Propositions 1 and 3: as the marginal benefit of effort vanishes for the fit candidate, the manager is soft for relatively low priors, which coincides with lower standards in the dynamic game than in the static game. Thus the manager's ability to commit causes not only the manager, but also the candidate (unfit or fit) to be better-off in the dynamic game.

[37] Information generation often prevents the uniqueness of the BNE; however, as the return to effort vanishes with increases in $\underline{\theta}_q$, uniqueness is recovered.

Now we turn our attention to the case in which the effect of the standard on the fit candidate's effort dominates.

Corollary 5 *Assume F.2-F.3 and C.1-C.2. Then, there exists* $\overline{\lambda} > 0$ *satisfying that for all* $\lambda \in [0, \overline{\lambda})$*, there exists* $\widetilde{\kappa}_G(\lambda) \in (\underline{\kappa}_C(\lambda), \overline{\kappa}_C(\lambda))$ *such that for all* $\kappa \in (\widetilde{\kappa}_G(\lambda), \overline{\kappa}_C(\lambda))$ *and SPNE* $(s_P^*, \theta_u^*(\cdot;\lambda), \theta_q^*)$ *of* $\Gamma_1(\lambda)$*, there is a BNE of* $\Gamma_0(\lambda)$ *Pareto dominated by* $(s_P^*, \theta_u^*(\cdot;\lambda), \theta_q^*)$*. Furthermore, if* $F_\theta(z, \overline{\theta}) = 0$ *for all* $z \in (0,1)$ *and (1) also holds at* $(0, \theta)$ *and* $(1, \theta)$ *for all* $\theta \in \Theta^\circ$*, then there exists* $\underline{\theta}_q{}' \in \Theta^\circ$ *satisfying that, for all* $\underline{\theta}_q \in \left(\underline{\theta}_q{}', \overline{\theta}\right]$*, the BNE of* $\Gamma_0(\lambda)$ *is unique.*

As in the previous result, Pareto dominance of commitment occurs when the manager sets soft standards —which now arise for relatively high κ.

7 A Mechanism Design Approach

In this section, we consider an alternative response by the manager to the pre-contractual informational asymmetries: designing a revelation mechanism.[38] By the Revelation Principle, we can focus on direct mechanisms that are truthful (i.e., that, in equilibrium, induce the candidate to reveal her true type). We restrict attention to mechanisms without monetary transfers. Thus, the mechanisms that we consider are described by a decision rule mapping each report (*unfit* or *fit*) to probabilities of outright rejection, outright hiring, and using a test with approval standard s to make the decision.

In the Appendix we show that, without loss of generality, the analysis can be restricted to the class of mechanisms in which: (i) any candidate who claims to be unfit is outright rejected with probability $p \in [0,1]$ and hired otherwise, and (ii) any candidate who claims to be fit is asked to take a test. Thus, we only consider truthful revelation mechanisms characterized by a duplet $(s, p) \in [0,1]^2$, where s is the standard applied to a candidate reporting to be *fit*, and p is the probability of outright rejection for a candidate reporting to be *unfit*.

The individual rationality constraint for the unfit candidate is redundant: rejecting the contract yields a loss of $1 \geq p$ for all $p \in [0,1]$. The same applies to the fit candidate: $1 \geq F(s, \underline{\theta}_q) \geq F(s, \theta_q^*(s)) + C_q(\theta_q^*(s))$ for all $s \in [0,1]$.

Incentive-compatibility requires $p \leq F(s, \theta_u^*(s)) + C_u(\theta_u^*(s))$ and $F(s, \theta_q^*(s)) + C_q(\theta_q^*(s)) \leq p$ for the unfit and fit candidate, respectively. The first restriction is binding, as the expected loss to the manager is decreasing in p, whereas the second is not, by Assumption C.1.

[38]We thank Roland Strausz for suggesting that we consider a mechanism design approach.

Let $V_M(s) := F(s, \theta_q^*(s)) - \kappa\,(F(s, \theta_u^*(s)) + C_u(\theta_u^*(s)))$ for all $s \in [0,1]$. Thus, if (s_M, p_M) is an optimal mechanism for the manager, then s_M solves $\min_{s\in[0,1]} V_M(s)$ and $p_M = F(s_M, \theta_u^*(s_M)) + C_u(\theta_u^*(s_M))$. Define

$$\underline{\kappa}_M := \inf_{s\in(0,1)} \left\{ \frac{F(s,\theta_q^*(s))}{F(s,\theta_u^*(s)) + C_u(\theta_u^*(s))} \right\} \text{ and } \overline{\kappa}_M := \sup_{s\in(0,1)} \left\{ \frac{1 - F(s,\theta_q^*(s))}{1 - F(s,\theta_u^*(s)) - C_u(\theta_u^*(s))} \right\},$$

and let

$$v_M(s;\theta) := \frac{d_q(s)}{f(s,\theta)} \tag{10}$$

for all $s \in [0,1]$ and $\theta \in \Theta$.

Let $\kappa \rightrightarrows \mathcal{S}_M^*(\kappa)$ be the correspondence mapping $\kappa \in (0,\infty)$ to the set of standards applied to candidates reporting to be fit in an optimal mechanism.[39]

Proposition 6 *Assume F.1-F.2 and C.1. The correspondence $\mathcal{S}_M^*$ is weakly increasing over $(0,\infty)$.*[40] *If (s_M, p_M) is an optimal mechanism, then, (i) $(s_M, p_M) = (0,0)$ for all $\kappa < \underline{\kappa}_M$, $(s_M, p_M) = (s, F(s,\theta_u^*(s)) + C_u(\theta_u^*(s)))$ for all $\kappa \in (\underline{\kappa}_M, \overline{\kappa}_M)$ for some $s \in (0,1)$ satisfying $v_M(s;\theta_u^*(s)) = \kappa$, and $(s_M, p_M) = (1,1)$ for all $\kappa > \overline{\kappa}_M$; and (ii) the manager strictly prefers the optimal mechanism to the equilibria of Γ_1 for all $\kappa \in (\underline{\kappa}_M, \overline{\kappa}_M)$.*

The manager is better-off using the optimal mechanism than simply committing to a standard, for all priors leading to an interior equilibrium standard, due to the higher probability of rejecting the unfit candidate. The proof of Proposition 6 reveals that, as in the pure information generation scenario of the game in which the manager commits to a standard, the optimal mechanism enlarges the range of priors for which screening is worthy.

Candidates are better-off with the revelation mechanism than under simple commitment to a standard if $s_M < s_P^*$ and worse-off if $s_M > s_P^*$. Provided that the pseudo likelihood ratio function of Γ_1, v, and $v_M(\cdot;\theta_u^*(\cdot))$ are both strictly increasing, $s_M < s_P^*$ if $v_M(s_P^*;\theta_u^*(s_P^*)) > v(s_P^*)$, for all $\kappa \in \mathcal{S}_M^{*-1}(0,1)$. Similarly, $s_M > s_P^*$ if $v_M(s_P^*;\theta_u^*(s_P^*)) < v(s_P^*)$.

Functions v and $v_M(\cdot;\theta_u^*(\cdot))$ differ in the same manner that v and $g(\cdot,\theta_u^*(\cdot),\underline{\theta}_q)$ differ in the pure information undermining setup: by the presence of the term $F_\theta(\cdot,\theta_u^*(\cdot))\frac{d\theta_u^*(\cdot)}{ds}$ in the denominator of v. Thus, if the problem is sufficiently well-behaved, so that both functions are well-defined in $[0,1]$,[41] for all $s \in (0,1)$ we have

[39] As in the game in which the manager simply commits to a hiring standard (see Section 4.1), the possibility of multiple equilibria for some knife-edge values of κ, in general, cannot be ruled out.

[40] Adopting the definition of weakly increasing correspondence introduced in Section 3.

[41] See the discussion in footnote 26.

$v(s) > (=,<) v_M(s; \theta_u^*(s))$ if $s < (=,>) \hat{s}_u$.

A sufficient condition for both v and $v_M(\cdot; \theta_u^*(\cdot))$ to be well-defined and strictly increasing is that the fit candidate's natural readiness and the unfit candidate's marginal cost are large. Given a triplet (F, C_u, C_q), let $\Gamma_1(\underline{\theta}_q, \lambda)$ be the game in which the manager commits to a standard, the fit candidate has a natural readiness $\underline{\theta}_q$, and the unfit candidate has a cost function $\lambda^{-1} C_u$ if $\lambda \in (0,1]$, or the game Γ_3 if $\lambda = 0$. Also, let $\hat{s}_u(\lambda)$ be the modularity-switch point of the unfit candidate with cost function $\lambda^{-1} C_u$ for all $\lambda \in (0,1]$, and $\underline{\kappa}_M(\underline{\theta}_q, \lambda)$ and $\overline{\kappa}_M(\underline{\theta}_q, \lambda)$, defined as $\underline{\kappa}_M$ and $\overline{\kappa}_M$, respectively, but with $\theta_u^*(\cdot; \lambda)$ instead of θ_u^*, for all $\underline{\theta}_q \in \Theta^\circ$ and $\lambda \in [0,1]$.

Proposition 7 *Suppose that the triplet (F, C_u, C_q) satisfies Assumptions F.2-F.3, C.1-C.2, $F_\theta(z, \overline{\theta}) = 0$ for all $z \in (0,1)$, and condition (1) also holds at $(0, \theta)$ and $(1, \theta)$ for all $\theta \in \Theta^\circ$. Consider the family of games $\Gamma_1(\underline{\theta}_q, \lambda)$ with $\underline{\theta}_q \in \Theta^\circ$ and $\lambda \in [0,1]$. Then, there exist $\underline{\underline{\theta}}_q \in \Theta^\circ$ and $\overline{\lambda} \in (0,1)$ such that $\underline{\theta}_q \in (\underline{\underline{\theta}}_q, \overline{\theta})$ and $\lambda \in (0, \overline{\lambda})$ imply that both the fit and unfit candidate prefer commitment to a standard over the optimal mechanism if and only if $\kappa \in (\underline{\kappa}_M(\underline{\theta}_q, \lambda), v(\hat{s}_u(\lambda)))$ and the optimal mechanism over commitment to a standard if and only if $\kappa \in (v(\hat{s}_u(\lambda)), \overline{\kappa}_M(\underline{\theta}_q, \lambda))$.*

Computations analogous to those leading to Proposition 1 yield that standards in the commitment setup are lower than in the optimal mechanism for low priors and higher for high priors. This is illustrated in Figure 4 for the example described in Section 5.3.[42] The economics behind Proposition 7, however, is very different from that in Proposition 1: with the optimal mechanism, at the margin, the manager ignores the effect of the standard on the unfit candidate's effort because, since $p_M = F(s_M, \theta_u^*(s_M)) + C_u(\theta_u^*(s_M))$, the optimal menu offsets changes in $F(\cdot, \theta_u^*(\cdot))$ with changes in $C_u(\theta_u^*(\cdot))$ (which, at the margin, are the same).

By Propositions 6 and 7, we have that under the assumptions of Proposition 7, the optimal mechanism Pareto Dominates the game $\Gamma_1(\underline{\theta}_q, \lambda)$ with $\underline{\theta}_q \in (\underline{\underline{\theta}}_q, \overline{\theta})$ and $\lambda \in (0, \overline{\lambda})$ if and only if the prior probability that the candidate is unfit is relatively high ($\kappa \in (v(\hat{s}_u(\lambda)), \overline{\kappa}_M(\underline{\theta}_q, \lambda))$).

[42] Under the assumptions of Proposition 7, if F has a neutral signal and information undermining (generation) dominates in Γ_1, then, the manager's deviations from the standard in the static game are in opposite (the same) direction under commitment and the optimal mechanism. When they are in the same direction, the deviation under the optimal mechanism is larger than under commitment, because of the buffering effect of the unfit candidate's effort in the latter.

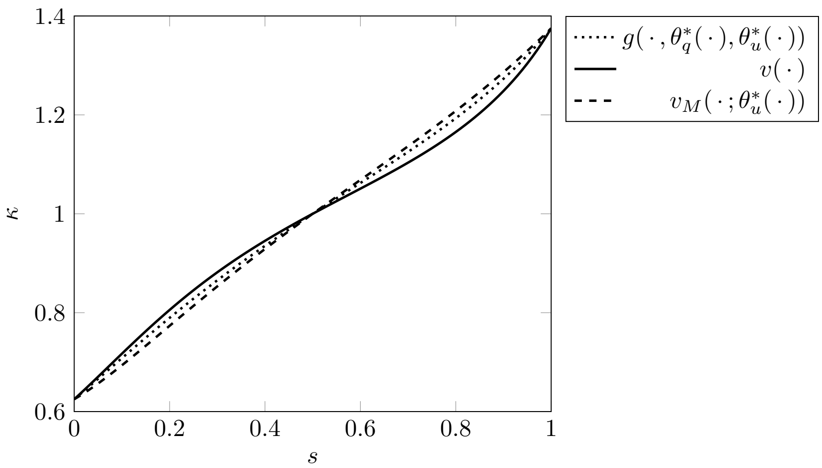

Figure 4: Example from Section 5.3 with $\lambda = \underline{\theta}_q = 0.5$: equilibrium standard in Γ_1 (solid line) and Γ_2 (dotted line), and the optimal mechanism standard (dashed line), for each prior κ.

8 Discussion

Optimal standards trade off classic statistical decision-making for management of information manipulation. Strategic complementarity between readiness and the standard develops in the submodular region of the domain of the signal distribution —i.e., for low standards that arise in equilibrium when agents have good prior prospects. Analogously strategic substitutability arises for agents with bad prior prospects. Thus, the decision maker often sets confirmative standards in problems dominated by information undermining and conservative standards in problems dominated by information generation. A revelation mechanism allows the decision maker to obtain a higher expected payoff than simple commitment to a standard.

Information manipulation by interested parties is ubiquitous. An application often discussed in the literature is the drug approval process by regulatory agencies such as the FDA or the ABPI (see, e.g., Li (2001), Henry and Ottaviani (2019)). As Li (2001) observes, "most of the evidence concerning effectiveness of a new drug is provided by its producer, not by the panelists." Pharmaceutical companies engage in a range of information manipulation practices, including hiding data, cherry-picking variables, manipulating experimental conditions, etc. (see, e.g., Goldacre (2014)). In the light of our results, a question that arises is whether regulatory agencies' approval standards are tilted in the right direction to manage information manipulation incentives.

For instance, our model predicts that, when information undermining is dominant, drugs with good prospects (low κ) should be subject to soft approval standards; namely, drugs with ex-post evidence marginally-negative expected values should be approved.[43] A natural choice for drugs having good prospects are those in the Breakthrough-Drug Designation (BDD) program of the FDA. As Darrow et al. (2018) show, trials following the nomination of many of the drugs in the BDD program have confirmed their good prospects, producing good results, and have been approved by the FDA. Nevertheless, trial results for some of the drugs in the program have shown little efficacy, even failing to meet customary standards (see Darrow et al. (2018), p. 1449). Yet, a number of such drugs have been approved by the FDA. Propositions 1 and 3 suggest that softening the standard has a positive side-effect of discouraging information undermining.[44] An empirical study of these issues is a subject of interest for future research.

As psychological screening has become widespread,[45] practitioners have emphasized the importance of assessing it properly (see, e.g., Dattner (2013), Caska (2020)). The assessment of screening tests, not only needs to take into account their performance in terms of wrongful hiring/rejection, but also, their effect on test-preparation incentives. While the performance of selection procedures is determined by many factors, practitioners should bare in mind the insights of our analysis. In particular, protocols more involved than plain tests, as those described in our mechanism design approach, may be advantageous for managers' hiring procedures.

Finally, in our analysis of commitment, the decision maker only uses standards of evidence as a tool to manage information manipulation. As argued above (in footnote 21), the use of approval standards in applications is widespread due to practical reasons. Our mechanism design approach illustrates that managing information manipulation can further benefit from other incentive schemes in the economics toolkit. Other possible approaches include mechanisms with transfers, manager's randomizations (probabilities of outright hiring/rejection), and hiring/rejection sets that are not monotone (i.e., not determined by a single standard) for both the commitment setup and revelation mechanisms. We leave for future research the analysis of these variations of the problem.

[43] Recall from footnote 15 that κ can be interpreted as capturing the weights of losses associated with wrong rejection and wrong acceptance.

[44] The BDD scheme was conceived to provide a "fast-track" approval process. The softening of standards that we refer to, however, is not related to the "fast-track" aspect of the program, but exclusively to the evidence documented in Darrow et al. (2018) on the approval of drugs that showed little efficacy in trials run after the drug was granted the designation.

[45] See SHL 2018 Global Assessment Trends Report `https://www.shl.com/en/assessments/trends/`.

Appendix: Proofs and Ancillary Material

Proofs and Ancillary Material of Section 2

Under Assumption F.2, $-F_\theta(z,\theta) := -\frac{\partial F(z,\theta)}{\partial\theta} > 0$ for all $z \in (0,1)$ and $\theta < \overline{\theta}$.

Claim 1 *Assume F.2. Then,* $F_\theta(z,\theta) < 0$ *for all* $z \in (0,1)$ *and* $\theta < \overline{\theta}$.

Proof. For all $z \in (0,1)$ and $\theta < \overline{\theta}$, we have $F(z,\theta) = \int_0^z e^{\ln f(z',\theta)} dz'$. Thus,

$$\begin{aligned} F_\theta(z,\theta) &= \int_0^z e^{\ln f(z',\theta)} \frac{1}{f(z',\theta)} \frac{\partial f(z',\theta)}{\partial\theta} dz' \\ &= \int_0^z e^{\ln f(z',\theta)} \left[\frac{1}{f(0,\theta)} \frac{\partial f(0,\theta)}{\partial\theta} + \int_0^{z'} \frac{\partial \frac{1}{f(z'',\theta)} \frac{\partial f(z'',\theta)}{\partial\theta}}{\partial z''} dz'' \right] dz' \\ &= \int_0^z f(z',\theta) \left[\frac{1}{f(0,\theta)} \frac{\partial f(0,\theta)}{\partial\theta} + \int_0^{z'} \frac{\partial^2 \ln f(z'',\theta)}{\partial z'' \partial\theta} dz'' \right] dz'. \end{aligned} \tag{11}$$

Since Assumption F.2 implies the MLRP, we know that $F_\theta(z,\theta) \leq 0$ for all $z \in (0,1)$ and $\theta < \overline{\theta}$. We now show that this inequality is indeed strict. If (11) is equal to zero for some $z \in (0,1)$ and $\theta < \overline{\theta}$, then $F_\theta(z',\theta) > 0$ for all $z' \in (z,1)$, as the term in the square brackets is increasing in z' and equals zero itself at most once. But $F_\theta(z',\theta) > 0$ contradicts FOSD (and hence the MLRP). ■

Assumptions F.1-F.2 guarantee the existence of a function $\widetilde{s} : [\underline{\theta}, \overline{\theta}) \to (0,1)$ separating the regions of D° where F is submodular and supermodular.

Remark 3 *Assume F.1-F.2. For all* $\theta < \overline{\theta}$ *there exists* $\widetilde{s}(\theta) \in (0,1)$ *such that*

$$\frac{\partial f(z,\theta)}{\partial\theta} \begin{cases} < 0 & \text{if } z < \widetilde{s}(\theta) \\ = 0 & \text{if } z = \widetilde{s}(\theta) \\ > 0 & \text{if } z > \widetilde{s}(\theta) \end{cases} \tag{12}$$

for all $z \in (0,1)$.[46]

Let $m(z,\theta) := \frac{1}{f(z,\theta)} \frac{\partial f(z,\theta)}{\partial\theta}$ for all $(z,\theta) \in D$.

Proof. By Claim 1, for all $z \in (0,1)$ and $\theta < \overline{\theta}$, we have that $\int_0^z \frac{\partial F_\theta(z',\theta)}{\partial z} dz' = F_\theta(z,\theta) < 0$, where the equality follows from the fact that $F_\theta(0,\theta) = 0$ for all $\theta < \overline{\theta}$. Thus there is $z' \in (0,z)$ such that $\frac{\partial F_\theta(z',\theta)}{\partial z} < 0$; and similarly, there is $z'' \in (z,1)$ such that $\frac{\partial F_\theta(z'',\theta)}{\partial z} > 0$. Therefore, $m(z',\theta) < 0$ and $m(z'',\theta) > 0$, and by continuity of m, there is $z''' \in (z',z'')$ such that $m(z''',\theta) = 0$.

[46] We allow for $F_\theta(z,\overline{\theta}) = 0$ for all $z \in [0,1]$, thus, it is possible that $\frac{\partial f(z,\overline{\theta})}{\partial\theta} = 0$ for all $z \in [0,1]$.

Indeed, z''' is the unique root of $m(\cdot,\theta)=0$ because Assumptions F.1 and F.2 imply that $m(\cdot,\theta)$ is strictly increasing for all $\theta<\overline{\theta}$:

$$\frac{\partial m(z,\theta)}{\partial z}=\frac{\partial^2 \ln f(z,\theta)}{\partial\theta\partial z}>0$$

for all $z\in(0,1)$ and $\theta<\overline{\theta}$. Thus, $\theta\mapsto\widetilde{s}(\theta)$ maps θ to the unique root of $m(\cdot,\theta)=0$, for all $\theta<\overline{\theta}$. ■

Let $s^*:\boldsymbol{\Theta}\times(0,\infty)\to[0,1]$ be the function mapping readiness profiles and priors $(\boldsymbol{\theta},\kappa)$ to the *ex-post optimal standard*; i.e., the standard minimizing the manager's expected loss, given the readiness pair $\boldsymbol{\theta}$. Define the *likelihood ratio function* $g:[0,1]\times\boldsymbol{\Theta}\to\mathbb{R}\cup\{\infty\}$ with $g(s,\boldsymbol{\theta}):=\frac{f(s,\theta_q)}{f(s,\theta_u)}$ for all $(s,\boldsymbol{\theta})\in[0,1]\times\boldsymbol{\Theta}$. By MLRP, $g(\cdot,\boldsymbol{\theta})$ is strictly increasing for all $\boldsymbol{\theta}\in\boldsymbol{\Theta}$. By F.1, $\text{sign}\left\{\frac{\partial V(s,\boldsymbol{\theta})}{\partial s}\right\}=\text{sign}\left\{g(s,\boldsymbol{\theta})-\kappa\right\}$ for all $(s,\boldsymbol{\theta})\in(0,1)\times\boldsymbol{\Theta}$. Thus, for all $\boldsymbol{\theta}\in\boldsymbol{\Theta}$, the optimal standard is

$$s^*(\boldsymbol{\theta};\kappa)=\begin{cases}0 & \text{if } 0<\kappa\le g(0,\boldsymbol{\theta})\\ s^*_{\boldsymbol{\theta},\kappa} & \text{if } g(0,\boldsymbol{\theta})<\kappa<g(1,\boldsymbol{\theta})\\ 1 & \text{if } g(1,\boldsymbol{\theta})\le\kappa,\end{cases}\tag{13}$$

where $s^*_{\boldsymbol{\theta},\kappa}$ is defined by $g(s^*_{\boldsymbol{\theta},\kappa},\boldsymbol{\theta})\equiv\kappa$ for all $\kappa\in(g(0,\boldsymbol{\theta}),g(1,\boldsymbol{\theta}))$. Since $g(\cdot,\boldsymbol{\theta})$ is strictly increasing, $s^*(\boldsymbol{\theta};\cdot)$ is weakly increasing for all $\boldsymbol{\theta}\in\boldsymbol{\Theta}$.

Proof of Lemma 1. By the Implicit Function Theorem, $\widetilde{s}$ is continuous, with $\frac{d\widetilde{s}(\theta)}{d\theta}=-\frac{\partial m(\widetilde{s}(\theta),\theta)}{\partial\theta}\left(\frac{\partial m(\widetilde{s}(\theta),\theta)}{\partial s}\right)^{-1}$ for all $\theta<\overline{\theta}$. In particular, $\frac{\partial m(\widetilde{s}(\theta),\theta)}{\partial s}>0$ and hence, $\frac{d\widetilde{s}(\theta)}{d\theta}$ is finite for all $\theta<\overline{\theta}$, by Assumptions F.1 and F.2.

Let $\hat{s}_u$ be a global maximizer θ^*_u. From Assumption C.1, $\hat{s}_u\in(0,1)$, and $\theta^*_u(\hat{s}_u)>\underline{\theta}$. Further, $\frac{d\theta^*_u(\hat{s}_u)}{ds}=0$ and hence, by (5), $\widetilde{s}(\theta^*_u(\hat{s}_u))=\hat{s}_u$. Indeed, $\hat{s}_u$ is the unique maximizer of θ^*_u: if $\hat{s}'_u\neq\hat{s}_u$ is another maximizer of θ^*_u, then $\widetilde{s}(\theta^*_u(\hat{s}_u))=\hat{s}'_u$, contradicting that $\widetilde{s}$ is a function.

Suppose there exists $s'\neq\hat{s}_u$ such that $\widetilde{s}(\theta^*_u(s'))=s'$. If s' is not a local extreme of θ^*_u, then s' is a tangency point between θ^*_u and the inverse of $\widetilde{s}$.[47] But this would imply $0=d\theta^*_u(s')/ds=(d\widetilde{s}(\theta^*_u(s'))/d\theta)^{-1}$, which leads to a contradiction because $\frac{d\widetilde{s}(\theta)}{d\theta}$ is finite for all $\theta<\overline{\theta}$. Furthermore, s' cannot be a local minimum of θ^*_u as this would imply that for some $\theta>\theta^*_u(s')$, there are $s''<s'<s'''$ with $\theta^*_u(s'')=\theta^*_u(s''')=\theta$ and such that $d\theta^*_u(s'')/ds<0$ and $d\theta^*_u(s''')/ds>0$, implying $\widetilde{s}(\theta)<s''<s'''<\widetilde{s}(\theta)$, a contradiction. Therefore s' can only be a local maximum of θ^*. But this would imply that there is a local minimum of θ^*_u in the interval $(\min\{s',\hat{s}_u\},\max\{s',\hat{s}_u\})$, contradicting that θ^*_u does not have local minima in

[47] The inverse of $\widetilde{s}$ then could be defined over an open interval containing s' because the tangency occurring under the working hypothesis would imply that $d\widetilde{s}(\theta^*_u(s'))/d\theta\neq 0$.

$(0, 1)$. We conclude that θ_u^* intersects $\widetilde{s}$ only once at $(\hat{s}_u, \theta_u^*(\hat{s}_u))$; hence, the thesis of the lemma follows immediately. $\square$

Formal analysis of data manipulation processes.

Sample cherry-picking. A pharmaceutical company runs an experiment aiming to get a new drug approved by a regulatory agency. The observed effect in the experiment is $x = \theta + u$, where u is independent of the effectiveness of the drug and is normally distributed, $u \sim N(0, \sigma^2)$. The average observed effect of an ineffective drug, θ_u, is 0 if the subjects' average age in the experiment is the same as the average age of the target population of the drug, $\overline{a}$. On average, the result of the experiment is decreasing on the average age of the individuals in the sample, a, according to a quadratic function, $\theta_u = (\overline{a} - a)^2$ for all $a \in [0, \overline{a}]$. The average performance in the experiment of an effective drug is uniformly superior for all ages by $\underline{\theta}_q > 0$. The costs of controlling the average age in the sample are $\frac{1}{2}c(\overline{a} - a)^4$, with $c > 0$. The regulatory agency's expected loss is given by (2). Although the range of x is $\mathbb{R}$, using a suitable transformation, we obtain a random variable whose distribution, along with the cost function of controlling the subjects' average age, define a problem that satisfies Assumptions F.1-F.2 and C.1.

From the above cost structure, we obtain $C_u(\theta_u) = \frac{1}{2}c\theta_u^2$ for all $\theta_u \in [0, \overline{a}^2]$. Similarly, $C_q(\theta_q) = \frac{1}{2}c(\theta_q - \underline{\theta}_q)^2$ for all $\theta_q \in [\underline{\theta}_q, \underline{\theta}_q + \overline{a}^2]$.

Let $z := \frac{1}{\pi} \arctan x + \frac{1}{2} \in [0, 1]$. Then,

$$F(z, \theta) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}\left(\frac{\tan y(z') - \theta}{\sigma}\right)^2} \pi(1 + \tan^2(y(z')))dz',$$

where $y(z') := \pi\left(z' - \frac{1}{2}\right)$ for all $z' \in [0, 1]$, and

$$\frac{\partial^2 F(z, \theta)}{\partial \theta^2} = \frac{1}{\sigma^3\sqrt{2\pi}} \int_0^z e^{-\frac{1}{2}\left(\frac{\tan y(z') - \theta}{\sigma}\right)^2} \pi(1 + \tan^2(y(z'))) \left[\left(\frac{\tan y(z') - \theta}{\sigma}\right)^2 - 1\right] dz'.$$

Direct computations show that F satisfies Assumptions F.1-F.2. Furthermore, for each θ, $\frac{\partial^2 F(z,\theta)}{\partial \theta^2}$ attains its minimum at $z(\theta) := \frac{1}{\pi}\arctan(\sigma + \theta) + \frac{1}{2}$; and direct computations reveal that $\frac{\partial^2 F(z(\theta),\theta)}{\partial \theta^2}$ does not depend on θ.[48] Thus, since $C_u'' = C_q'' = c$, Assumption C.1 holds if $c + \frac{\partial^2 F(z(0),0)}{\partial \theta^2} > 0$.

Hidden data disposal. Consider a pharmaceutical company that runs experiments to get a new drug approved by a regulatory agency. The outcome of an experiment,

[48]It can be shown that $\frac{\partial F^2(z(\theta),\theta)}{\partial \theta^2}$ is equal to σ^{-2} times the probability that $x < \theta + \sigma$, times the conditional expected value of $\frac{(x-\theta)^2}{\sigma^2} - 1$ given $x < \theta + \sigma$ (which does not depend on θ as changes in θ only shift the distribution of x to the left or right).

$z \in [0,1]$, is distributed according to the atomless distribution function H with density $h > 0$, if the drug is ineffective, and (for simplicity) according to the distribution H^2, if the drug is effective. The company can run additional tests, and select the best outcome realization. The cost of running n tests (in total) is $\frac{1}{2}c(n-1)^2$ for some $c > 0$ and at least one test has to be run. The discreteness of the number of tests (which determines readiness in this setup) prevents this model from satisfying Assumptions F.1-F.2 and C.1. Nevertheless, it is easy to show that the economically substantial properties of our model are satisfied in this setup. Thus, the core qualitative aspects of our analysis, with suitable adjustments to account for the discreteness of readiness, are featured within this setup as well.

Proofs and Ancillary Material for Section 3

Static Game. Now we consider the imperfect information *static game* Γ_0 between the manager and the candidate, who simultaneously choose the standard and readiness, respectively. Their expected loss are given by (2) and (3), respectively. A *pure strategy Bayesian Nash equilibrium* of Γ_0 is a triplet $(s^*_{NE}, \theta_{uNE}, \theta_{qNE}) \in D \times \Theta_q$, with $s^*_{NE} = s^*(\theta_{uNE}, \theta_{qNE}; \kappa)$, $\theta_{uNE} = \theta^*_u(s^*_{NE})$, and $\theta_{qNE} = \theta^*_q(s^*_{NE})$, and hence, satisfying equations (13) and (4).

There exists a BNE with $s^*_{NE} \in (0,1)$ and, hence, $g(s^*_{NE}, \theta^*_u(s^*_{NE}), \theta^*_q(s^*_{NE})) = \kappa$, for all $\kappa \in \left(\inf_{s\in(0,1)} g(s, \theta^*_u(s), \theta^*_q(s)), \sup_{s\in(0,1)} g(s, \theta^*_u(s), \theta^*_q(s))\right)$. On the other hand, $(0, \underline{\theta}, \underline{\theta}_q)$ is the unique BNE if $\kappa < \inf_{s\in(0,1)} g(s, \theta^*_u(s), \theta^*_q(s))$, and $(1, \underline{\theta}, \underline{\theta}_q)$ is the unique BNE if $\kappa > \sup_{s\in(0,1)} g(s, \theta^*_u(s), \theta^*_q(s))$.

Proof of Lemma 2. Observe that

$$\begin{aligned}
\frac{d\mathcal{V}(s)}{ds} &= f(s, \theta^*_u(s)) \left(g(s, \theta^*_u(s), \theta^*_q(s)) - \kappa\right) + F_\theta(s, \theta^*_q(s)) \frac{d\theta^*_q(s)}{ds} - \kappa F_\theta(s, \theta^*_u(s)) \frac{d\theta^*_u(s)}{ds} \\
&= f(s, \theta^*_u(s)) \left(g(s, \theta^*_u(s), \theta^*_q(s)) - \kappa\right) + F_\theta(s, \theta^*_u(s)) \frac{d\theta^*_u(s)}{ds} \left(r(s) - \kappa\right)
\end{aligned}$$

for all $s \in (0,1) \backslash \{\hat{s}_u\}$. The manager is soft (harsh) in an equilibrium with standard $s^*_P \in (0,1)$ if $g(s^*_P, \theta^*_u(s^*_P), \theta^*_q(s^*_P)) < (>) \kappa$. Thus, parts (i)-(ii) and (iii)-(iv) follow from the first and second equalities, respectively, using Claim 1 and Remark 1.[49]
□

[49] A more intuitive proof for parts (i) and (ii) was provided in the text.

Proofs and Ancillary Material of Section 4

Proof of Lemma 3. We prove that $\mathcal{S}^*$ is weakly increasing using an indirect argument. Consider $\kappa' > \kappa$, $s \in \mathcal{S}^*(\kappa)$, and $s' \in \mathcal{S}^*(\kappa')$. Then,

$$\begin{aligned} F(s,\underline{\theta}_q) - \kappa F(s,\theta_u^*(s)) &\leq F(s',\underline{\theta}_q) - \kappa F(s',\theta_u^*(s')) \\ F(s',\underline{\theta}_q) - \kappa' F(s',\theta_u^*(s')) &\leq F(s,\underline{\theta}_q) - \kappa' F(s,\theta_u^*(s)). \end{aligned}$$

Adding these inequalities yields $(\kappa' - \kappa)\left(F(s,\theta_u^*(s)) - F(s',\theta_u^*(s'))\right) \leq 0$, which implies $F(s,\theta_u^*(s)) \leq F(s',\theta_u^*(s'))$. Now suppose $s' < s$; since densities are strictly positive, we have $F(s',\underline{\theta}_q) < F(s,\underline{\theta}_q)$. Thus, $F(s',\underline{\theta}_q) - \kappa F(s',\theta_u^*(s')) < F(s,\underline{\theta}_q) - \kappa F(s,\theta_u^*(s))$, contradicting that s is the equilibrium standard for κ.

Now we show that $\mathcal{S}^*(\underline{\kappa}) = \{0\}$. Notice that $V(0,\underline{\theta},\underline{\theta}_q;\underline{\kappa}) < V(s,\underline{\theta},\underline{\theta}_q;\underline{\kappa}) \leq V(s,\theta_u^*(s),\underline{\theta}_q;\underline{\kappa})$, for all $s > 0$, where the strict inequality follows from the fact that $s^*(\underline{\theta},\underline{\theta}_q;\underline{\kappa}) = 0$, and the weak inequality follows from FOSD.

Since $\mathcal{S}^*$ is weakly increasing, we conclude $\mathcal{S}^*(\kappa) = \{0\}$ for all $\kappa \in (0,\underline{\kappa}]$. An analogous argument proves that $\mathcal{S}^*(\kappa) = \{1\}$ for all $\kappa \in [\overline{\kappa},\infty)$.

Finally, we show that $\mathcal{S}^*$ is strictly increasing over $\mathcal{S}^{*-1}(0,1)$. Notice that for all $\kappa \in \mathcal{S}^{*-1}(0,1)$, $s \in \mathcal{S}^*(\kappa)$ only if the right hand side of (7) is equal to 0. Thus, s can only be an element of $\mathcal{S}^*(\kappa)$ for only one $\kappa \in \mathcal{S}^{*-1}(0,1)$. $\square$

Proof of Proposition 1. For any game Γ_2, let $\widetilde{\kappa}_U := \sup\{\kappa \in (0,\infty) : \sup \mathcal{S}^*(\kappa) \leq \hat{s}_u\}$, where $\hat{s}_u$ is the modularity-switch point.

Part 1. We first prove that $\widetilde{\kappa}_U < \overline{\kappa}$:

Case 1. $\overline{\kappa} = \infty$: For any $\kappa \in (0,\infty)$, we have that

$$\min_{s\in[0,\hat{s}_u]} \left\{V(1,\underline{\theta},\underline{\theta}_q;\kappa) - V(s,\theta_u^*(s),\underline{\theta}_q;\kappa)\right\} = \min_{s\in[0,\hat{s}_u]} \left\{1 - F(s,\underline{\theta}_q) - \kappa(1 - F(s,\theta_u^*(s)))\right\}.$$

This expression is negative for a large enough κ°, thus $s \notin \mathcal{S}^*(\kappa^\circ)$ for all $s \in [0,\hat{s}_u]$. Since $\mathcal{S}^*$ is weakly increasing (Lemma 3), $s \notin \mathcal{S}^*(\kappa')$ for all $s \in [0,\hat{s}_u]$ and $\kappa' > \kappa^\circ$. Thus, $\widetilde{\kappa}_U \leq \kappa^\circ < \infty$.

Case 2. $\overline{\kappa} < \infty$: From Lemma 3, $\mathcal{S}^*(\overline{\kappa}) = \{1\}$. In particular, $V(1,\underline{\theta},\underline{\theta}_q;\overline{\kappa}) < \min_{s\in[0,\hat{s}_u]} V(s,\theta_u^*(s),\underline{\theta}_q;\overline{\kappa})$. Notice that $V(s,\theta,\underline{\theta}_q;\cdot)$ is continuous over $(0,\infty)$, for all $(s,\theta) \in [0,1]\times\Theta$, and so it is $\min_{s\in[0,\hat{s}_u]} V(s,\theta_u^*(s),\underline{\theta}_q;\cdot)$, by the Maximum Theorem. Thus, for small enough $\delta > 0$, we have $V(1,\underline{\theta},\underline{\theta}_q;\overline{\kappa}-\delta) < \min_{s\in[0,\hat{s}_u]} V(s,\theta_u^*(s),\underline{\theta}_q;\overline{\kappa}-\delta)$. Therefore, $\sup \mathcal{S}^*(\overline{\kappa}-\delta) > \hat{s}_u$ and since $\mathcal{S}^*$ is weakly increasing (Lemma 3), we conclude that $\widetilde{\kappa}_U \leq \overline{\kappa} - \delta < \overline{\kappa}$.

Part 2. Now we show that the manager is ex-post efficient for all $\kappa \in [\overline{\kappa},\infty)$: From Lemma 3, $\mathcal{S}^*(\kappa) = \{1\}$ for all $\kappa \geq \overline{\kappa}$. Recall that $\theta_u^*(1) = \underline{\theta}$. From (13),

$s^*(\underline{\theta}, \underline{\theta}_q; \kappa) = 1$ for all $\kappa \geq \overline{\kappa}$. Thus, the manager is ex-post efficient for all $\kappa \geq \overline{\kappa}$.
Part 3. Now we show that the manager is harsh for all $\kappa \in (\widetilde{\kappa}_U, \overline{\kappa})$: consider any $\kappa \in (\widetilde{\kappa}_U, \overline{\kappa})$ and $s \in \mathcal{S}^*(\kappa)$; since $\mathcal{S}^*$ is weakly increasing over $(0, \infty)$ and strictly increasing over $\mathcal{S}^{*-1}(0,1)$ (by Lemma 3), we have that $s \in (\hat{s}_u, 1]$. Lemmata 1 and 2 (see footnote 27) imply that the manager is harsh if $s \in (\hat{s}_u, 1)$. And if $s = 1$ the manager is harsh because $s^*(\underline{\theta}, \underline{\theta}_q; \kappa) < 1$ for $\kappa < \overline{\kappa}$.

Noting that $\widetilde{\kappa}_U = \inf\{\kappa \in (0,\infty) : \inf \mathcal{S}^*(\kappa) \geq \hat{s}_u\}$, an argument analogous to that of Part 1 shows that $\widetilde{\kappa}_U > \underline{\kappa}$. Similarly, arguments analogous to those in Parts 2 and 3 yield that the manager is ex-post efficient for all $\kappa \in (0, \underline{\kappa}]$ and soft for all $\kappa \in (\underline{\kappa}, \widetilde{\kappa}_U)$, respectively. □

Remark 4 *Assume that F satisfies F.2 and F.3, C_u satisfies Assumption C.1 (i)-(iv) and C.2, and (1) also holds at $(0, \theta)$ and $(1, \theta)$ for all $\theta \in \Theta^\circ$. Then, there exists $\overline{\lambda} > 0$ such that the game Γ_2 defined by F and the cost function $\lambda^{-1} C_u$ has a strictly increasing pseudo likelihood ratio function v, for all $\lambda \in (0, \overline{\lambda})$.*

Proof. Let $\theta_u^*(\cdot; \lambda)$ be the unfit candidate's best response for the cost function $\lambda^{-1} C_u$ if $\lambda > 0$ and $\theta_u^*(\cdot; \lambda) = \underline{\theta}$ if $\lambda = 0$. If $\lambda > 0$, the derivative of $F(\cdot, \theta_u^*(\cdot; \lambda))$ is

$$d_u(s; \lambda) := f(s, \theta_u^*(s;\lambda)) - F_\theta(s, \theta_u^*(s;\lambda)) \frac{\partial f(s, \theta_u^*(s;\lambda))}{\partial \theta} \left(\frac{C_u''(\theta_u^*(s;\lambda))}{\lambda} + \frac{\partial^2 F(s, \theta_u^*(s;\lambda))}{\partial \theta^2} \right)^{-1}$$

for all $s \in [0, 1]$. Since $f(\cdot, \underline{\theta}) > 0$, by the Maximum Theorem, there exists $\overline{\lambda}_1 > 0$ such that for all $\lambda \in (0, \overline{\lambda}_1)$, we have that $\min_{s\in[0,1]} \{d_u(s; \lambda)\} > 0$. Thus, for all $\lambda \in (0, \overline{\lambda}_1)$, $v' > 0$ is equivalent to

$$\min_{s\in[0,1]} \left\{ \frac{1}{f(s, \underline{\theta}_q)} \frac{\partial f(s, \underline{\theta}_q)}{\partial s} - \frac{1}{d_u(s;\lambda)} \frac{d\,(d_u(s;\lambda))}{ds} \right\} > 0.$$

Indeed,

$$\lim_{\lambda\to 0} \min_{s\in[0,1]} \left\{ \frac{1}{f(s,\underline{\theta}_q)} \frac{\partial f(s,\underline{\theta}_q)}{\partial s} - \frac{1}{d_u(s;\lambda)} \frac{d(d_u(s;\lambda))}{ds} \right\} = \min_{s\in[0,1]} \left\{ \frac{1}{f(s,\underline{\theta}_q)} \frac{\partial f(s,\underline{\theta}_q)}{\partial s} - \frac{1}{f(s,\underline{\theta})} \frac{\partial f(s,\underline{\theta})}{\partial s} \right\} > 0,$$

where the inequality is guaranteed by (1). Thus, there exists $\overline{\lambda} > 0$ such that for all $\lambda \in (0, \overline{\lambda})$, we have $v' > 0$. ■

Proof of Remark 2. Part (i) is direct, so we proceed directly to prove part (ii):

Case 1. Suppose $d_u(s) > 0$ over $(0, 1)$. For all $s \in (\underline{s}, \overline{s})$, if s is a critical point of $V(\cdot, \theta_u^*(\cdot), \underline{\theta}_q)$, then s is a local maximum and hence, $s \notin \mathcal{S}^*(\kappa)$ for any $\kappa \in (0, \infty)$. Let $\kappa^* := \sup\{\kappa \in (0,\infty) : \sup \mathcal{S}^*(\kappa) \leq \underline{s}\}$ and $\kappa_* := \inf\{\kappa \in (0,\infty) : \inf \mathcal{S}^*(\kappa) \geq \overline{s}\}$.

We observe that $\kappa^* = \kappa_*$: if $\kappa^* < \kappa_*$, then for all $\kappa \in (\kappa^*, \kappa_*)$ we have that $\mathcal{S}^*(\kappa) \cap (\underline{s}, \overline{s}) \neq \emptyset$, contradicting that $s \in (\underline{s}, \overline{s})$ implies that $s \notin \mathcal{S}^*(\kappa)$ for any $\kappa \in (0, \infty)$. On the other hand, if $\kappa^* > \kappa_*$, then for any $\kappa \in (\kappa_*, \kappa^*)$, $\sup \mathcal{S}^*(\kappa) \leq \underline{s}$ and $\inf \mathcal{S}^*(\kappa) \geq \overline{s}$, a contradiction.

Thus, for all $\kappa' < \kappa^*$ we have $\sup \mathcal{S}^*(\kappa') \leq \underline{s}$, and for all $\kappa'' > \kappa^*$ we have $\inf \mathcal{S}^*(\kappa'') \geq \overline{s}$. Hence the thesis holds for $\kappa = \kappa^*$ and all $\delta \in (0, \overline{s} - \underline{s})$.

Case 2. Suppose $d_u(s) \leq 0$ for some $s \in (0,1)$. Then, for all $\kappa \in (0, \infty)$, we have $dV(\cdot, \theta_u^*(\cdot), \underline{\theta}_q; \kappa)/ds > 0$ over $(s - \varepsilon, s + \varepsilon)$ for some $\varepsilon \in (0, \min\{s, 1 - s\})$. Thus, $s \notin \mathcal{S}^*(\kappa)$ for any $\kappa \in (0, \infty)$. Analogously to the argument in Case 1, we can define $\kappa'^* := \sup\{\kappa \in (0, \infty) : \sup \mathcal{S}^*(\kappa) \leq s - \varepsilon\}$ and $\kappa'_* := \inf\{\kappa \in (0, \infty) : \inf \mathcal{S}^*(\kappa) \geq s + \varepsilon\}$. The rest of the argument is analogous to Case 1, leading to the statement that thesis holds for $\kappa = \kappa'^*$ and all $\delta \in (0, 2\varepsilon)$. $\square$

Quasi-Symmetric Distributions. For each $\theta \in \Theta^\circ$, we implicitly define the function $s \mapsto s_f(\cdot, \theta)$ by $F_\theta(s, \theta) = F_\theta(s_f(s, \theta), \theta)$, with $s_f(s, \theta) \neq s$ for all $s \in [0,1] \setminus \{\widetilde{s}(\theta)\}$, and $s_f(\widetilde{s}(\theta), \theta) = \widetilde{s}(\theta)$. The continuity of F_θ, $F_\theta(0, \theta) = F_\theta(1, \theta) = 0$, and $\frac{\partial F_\theta(s,\theta)}{\partial s} < (>) 0$ for all $s \in (0, \widetilde{s}(\theta))$ $(s \in (\widetilde{s}(\theta), 1))$ guarantee that s_f is well defined. If $s_f(s, \theta)$ does not depend on θ, then F is QS.

Claim 2 *Assume F.1-F.2. If F is QS, then: (i) F has a neutral signal $s^\star$, (ii) $\theta_u^*(s) = \theta_u^*(s_f(s))$ for all $s \in (0,1)$ and all cost function C_u satisfying conditions (i)-(iv) in Assumption C.1, (iii) $F(s, \theta) - F(s_f(s), \theta) = F(s, \theta') - F(s_f(s), \theta')$ for all $\theta, \theta' \in \Theta^\circ$ and for all $s \in [0,1]$; and (iv) $V(s, \theta_u^*(s), \underline{\theta}_q; 1) = V(s_f(s), \theta_u^*(s_f(s)), \underline{\theta}_q; 1)$ for all $s \in [0,1]$ and all cost function C_u satisfying conditions (i)-(iv) in Assumption C.1.*

Proof. For (i), consider a QS distribution F and the working hypothesis: $\widetilde{s}(\theta) \neq \widetilde{s}(\theta')$ for some $\theta, \theta' \in \Theta^\circ$. Since $\widetilde{s}(\theta)$ is the only fixed point of $s_f(\cdot, \theta)$, there exists $s' \neq \widetilde{s}(\theta')$ such that $F_\theta(\widetilde{s}(\theta'), \theta) = F_\theta(s', \theta)$. But then, since F is QS, $F_\theta(\widetilde{s}(\theta'), \theta') = F_\theta(s', \theta')$, contradicting that $\widetilde{s}(\theta')$ is the only fixed point of $s_f(\cdot, \theta')$. Thus, $\widetilde{s}$ is constant and hence F has a neutral signal.

Statement (ii) is immediate from (4). Statement (iii) follows from

$$
\begin{aligned}
F(s,\theta) - F(s_f(s),\theta) &= \int_{\underline{\theta}}^{\theta} F_\theta(s,t)dt + F(s,\underline{\theta}) - \int_{\underline{\theta}}^{\theta} F_\theta(s_f(s),t)dt - F(s_f(s),\underline{\theta}) \\
&= F(s,\underline{\theta}) - F(s_f(s),\underline{\theta})
\end{aligned}
$$

for all $s \in [0,1]$ and $\theta \in \Theta$. Finally, statement (iv) follows from (ii) and (iii), setting $\theta = \underline{\theta}_q$ and $\theta' = \theta_u^*(s)$. ■

We provide three simple properties on the functional form of F that are sufficient for QS. The first one is $\frac{\partial^2 F(z,\theta)}{\partial\theta^2} = 0$ for all $(z,\theta) \in (0,1) \times \Theta^\circ$. A distribution in the RD family satisfies this property if γ is an affine transformation of θ (see Example 2). The second property is separability of F_θ: $F_\theta(z,\theta) = \alpha(z)\beta(\theta)$ for all $z \in (0,1)$ and $\theta \in \Theta^\circ$, for some real functions $\alpha : (0,1) \to \mathbb{R}$ and $\beta : \Theta^\circ \to \mathbb{R}$. The RD family also satisfies this property with, e.g., $\alpha(z) = \int_0^z f_1(z')dz' - \int_0^z f_0(z')dz'$ and $\beta(\theta) = \gamma'(\theta)$. The third property is rotational symmetry of the off-diagonal terms of the Hessian of F: $\frac{\partial f(z,\theta)}{\partial\theta} = -\frac{\partial f(1-z,\theta)}{\partial\theta}$ for all $z \in (0, \frac{1}{2})$ and $\theta \in \Theta^\circ$, with $s_f(s,\theta) = 1 - s$ for all $s \in (0,1)$ and $\theta \in \Theta^\circ$.[50]

Proof of Proposition 3: In the sequel, when convenient, we make explicit the dependence of d_q, v, r, and θ_q^* on the natural readiness of fit candidates, so, instead of writing $d_q(s)$, $v(s)$, $r(s)$, and $\theta_q^*(s)$, we write $d_q(s;\underline{\theta}_q)$, $v(s;\underline{\theta}_q)$, $r(s;\underline{\theta}_q)$ and $\theta_q^*(s;\underline{\theta}_q)$, respectively, for all $s \in [0,1]$ and $\underline{\theta}_q \in \Theta^\circ$. The proof hinges on the following lemmata:

Lemma 4 *Assume F.2, F.3, C.1, C.2, and $F_\theta(s,\overline{\theta}) = 0$ for all $s \in (0,1)$. Then, there exists $\underline{\underline{\theta}}_q \in \Theta^\circ$ such that for all $\underline{\theta}_q \in \left(\underline{\underline{\theta}}_q, \overline{\theta}\right)$, $d_q(s;\underline{\theta}_q) > 0$ for all $s \in [0,1]$.*

Proof. The Maximum Theorem, Assumptions F.3 and C.1-C.2 guarantee that $\min_{s\in[0,1]} d_q(s;\underline{\theta}_q)$ varies continuously with $\underline{\theta}_q$. Further, $F_\theta(s,\overline{\theta}) = 0$ for all $s \in [0,1]$ and, from the hypothesis, $\min_{s\in[0,1]} f(s,\overline{\theta}) > 0$. Thus, there exists $\underline{\underline{\theta}}_q \in \Theta^\circ$ such that $\min_{s\in[0,1]} d_q(s;\underline{\theta}_q) > 0$ for all $\underline{\theta}_q \in \left(\underline{\underline{\theta}}_q, \overline{\theta}\right)$. ■

Lemma 5 *Assume F.2-F.3, C.1-C.2, and $F_\theta(s,\overline{\theta}) = 0$ for all $s \in (0,1)$. Then, there exists $\underline{\underline{\theta}}_q \in \Theta^\circ$ such that for all $\underline{\theta}_q \in \left(\underline{\underline{\theta}}_q, \overline{\theta}\right)$, there exists $\hat{s}(\underline{\theta}_q) \in (0,1)$ such that, if $(s_P^*, \theta_u^*, \theta_q^*)$ is a SPNE of Γ_1, then the manager is soft if $s_P^* \in (0, \hat{s}(\underline{\theta}_q))$ and harsh if $s_P^* \in (\hat{s}(\underline{\theta}_q), 1)$.*

Proof. First, direct computations yield

$$\lim_{(s,\underline{\theta}_q)\to(s_0,\overline{\theta})} \frac{dv(s;\underline{\theta}_q)}{ds} > \lim_{(s,\underline{\theta}_q)\to(s_0,\overline{\theta})} \frac{dg(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q))}{ds} \tag{14}$$

for $s_0 = 0, 1$. By Assumption F.3, $v(0;\underline{\theta}_q) = \underline{\kappa}(\underline{\theta}_q)$ and $v(1;\underline{\theta}_q) = \overline{\kappa}(\underline{\theta}_q)$, thus there exist $0 < \delta_1 < \delta_2 < 1$ and $\underline{\underline{\theta}}_{q_1} \in \Theta^\circ$ such that

$$v(s;\underline{\theta}_q) \begin{cases} > g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s \in (0,\delta_1) \\ < g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s \in (\delta_2, 1) \end{cases}$$

[50]Notice that $F_\theta(s,\theta) = \int_0^s \frac{\partial F_\theta(z,\theta)}{\partial z}dz = -\int_{1-s}^1 \frac{\partial F_\theta(z,\theta)}{\partial z}dz = F_\theta(1-s,\theta)$ for all $(s,\theta) \in (0,1) \times \Theta^\circ$.

for all $\underline{\theta}_q \in \left(\underline{\theta}_{q_1}, \overline{\theta}\right)$.

Second, by Assumption F.2, Lemma 1, and direct computations, we have

$$\lim_{(s,\underline{\theta}_q)\to(\hat{s}_u,\overline{\theta})} \frac{dv(s;\underline{\theta}_q)}{ds} < \lim_{(s,\underline{\theta}_q)\to(\hat{s}_u,\overline{\theta})} \frac{dg(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q))}{ds}. \tag{15}$$

Since $v(\hat{s}_u;\overline{\theta}) = g(\hat{s}_u, \theta_u^*(\hat{s}_u), \overline{\theta})$, there exist $\underline{\theta}_{q_2} \in \Theta^\circ$, $\delta_3 \in (\delta_1, \hat{s}_u)$, and $\delta_4 \in (\hat{s}_u, \delta_2)$ such that, for all $\underline{\theta}_q \in \left(\underline{\theta}_{q_2}, \overline{\theta}\right)$, there exists $\hat{s}(\underline{\theta}_q) \in (\delta_3, \delta_4)$ satisfying

$$v(s;\underline{\theta}_q) \begin{cases} > g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s \in (\delta_3, \hat{s}(\underline{\theta}_q)) \\ = g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s = \hat{s}(\underline{\theta}_q) \\ < g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s \in (\hat{s}(\underline{\theta}_q), \delta_4). \end{cases}$$

Third, in game Γ_2, for all $s \in [\delta_1, \delta_3]$, either $v(s;\overline{\theta}) > g(s, \theta_u^*(s), \overline{\theta})$ or $d_u(s) \leq 0$, and $v(s;\overline{\theta}) < g(s, \theta_u^*(s), \overline{\theta})$, for all $s \in [\delta_4, \delta_2]$. Thus, if $d_u > 0$ on $[\delta_1, \delta_3]$, then

$$v(s;\underline{\theta}_q) \begin{cases} > g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s \in [\delta_1, \delta_3] \\ < g(s,\theta_u^*(s),\theta_q^*(s;\underline{\theta}_q)) & \text{if } s \in [\delta_4, \delta_2] \end{cases}$$

for all large enough $\underline{\theta}_q \in \Theta^\circ$. We used Weierstrass' Theorem to establish that the difference between $v(s;\overline{\theta})$ and $g(s, \theta_u^*(s), \overline{\theta})$ is strictly greater than zero for all $s \in [\delta_1, \delta_3]$, and the Maximum Theorem to establish that the difference between $v(s;\underline{\theta}_q)$ and $g(s, \theta_u^*(s), \theta_q^*(s;\underline{\theta}_q))$ is strictly greater than zero for all $s \in [\delta_1, \delta_3]$, for $\underline{\theta}_q$ close enough to $\overline{\theta}$. An analogous argument applies for the interval $[\delta_4, \delta_2]$. On the other hand, if $d_u(s) < (=) 0$ for some $s \in [\delta_1, \delta_3]$, then by Lemma 4, $v(s;\underline{\theta}_q)$ is negative (not defined) for large enough $\underline{\theta}_q$.

Hence $v(\cdot;\underline{\theta}_q)$ and $g(\cdot, \theta_u^*(\cdot), \theta_q^*(\cdot;\underline{\theta}_q))$ cannot cross over $[\delta_1, \delta_3]$ or $[\delta_4, \delta_2]$ for all $\underline{\theta}_q \in \left(\underline{\theta}_{q_3}, \overline{\theta}\right)$, for a large enough $\underline{\theta}_{q_3} \in \Theta^\circ$.

Therefore, for all $\underline{\theta}_q > \underline{\underline{\theta}_q} := \max\left\{\underline{\theta}_{q_1}, \underline{\theta}_{q_2}, \underline{\theta}_{q_3}\right\}$, $\hat{s}(\underline{\theta}_q)$ is the only root of $v(\cdot;\underline{\theta}_q) - g(\cdot, \theta_u^*(\cdot), \theta_q^*(\cdot;\underline{\theta}_q))$ over $(0,1)$, and for any equilibrium $(s_P^*, \theta_q^*, \theta_u^*)$, we have $v(s_P^*;\underline{\theta}_q) > (<) g(s_P^*, \theta_u^*(s_P^*), \theta_q^*(s_P^*;\underline{\theta}_q))$ if $s_P^* < (>) \hat{s}(\underline{\theta}_q)$. Thus, the thesis of the lemma holds. ■

Lemma 6 *Assume F.2, F.3, C.1-C.2, and* $F_\theta(s, \overline{\theta}) = 0$ *for all* $s \in (0,1)$*. Then, there exists* $\underline{\underline{\theta}_q} \in \Theta^\circ$ *such that for all* $\underline{\theta}_q \in \left(\underline{\underline{\theta}_q}, \overline{\theta}\right)$*, the thesis of Lemma 3 holds in* Γ_1.

Proof. From Lemma 4, $F(\cdot, \theta_q^*(\cdot;\underline{\theta}_q))$ is strictly increasing for high enough $\underline{\theta}_q$. Thus, the argument showing that $\mathcal{S}^*$ is increasing in the proof of Lemma 3 applies here too.

We prove that $\mathcal{S}^*(\underline{\kappa}(\underline{\theta}_q)) = \{0\}$ (the proof of the statement $\mathcal{S}^*(\overline{\kappa}(\underline{\theta}_q)) = \{1\}$ is analogous). First notice that

$$\lim_{(s,\underline{\theta}_q)\to(0,\bar{\theta})} \frac{dv(s;\underline{\theta}_q)}{ds} = \lim_{s\to 0} \frac{dv(s;\bar{\theta})}{ds} > \lim_{s\to 0} \frac{dg(s,\theta_u^*(s),\bar{\theta})}{ds} > 0. \tag{16}$$

Thus, there exists $\underline{\theta}_{q_1} \in \Theta^\circ$ and $\delta > 0$ such that for all $\underline{\theta}_q \in \left(\underline{\theta}_{q_1}, \bar{\theta}\right)$, $\mathcal{V}(0;\underline{\kappa}(\underline{\theta}_q),\underline{\theta}_q) < \mathcal{V}(s;\underline{\kappa}(\underline{\theta}_q),\underline{\theta}_q)$ for all $s \in (0,\delta)$, where $\mathcal{V}(\cdot;\kappa,\underline{\theta}_q)$ is the expected loss to the manager $\mathcal{V}(s)$ for prior κ when the natural readiness of fit candidates is $\underline{\theta}_q$.

Second, from Lemma 3 and Assumption F.3, we know that $\lim_{\underline{\theta}_q\to\bar{\theta}} \mathcal{V}(0;\underline{\kappa}(\underline{\theta}_q),\underline{\theta}_q) < \lim_{\underline{\theta}_q\to\bar{\theta}} \mathcal{V}(s;\underline{\kappa}(\underline{\theta}_q),\underline{\theta}_q)$ for all $s > 0$. Let $\widetilde{\mathcal{V}}(\cdot,\underline{\theta}_q) := \mathcal{V}(\cdot;\underline{\kappa}(\underline{\theta}_q),\underline{\theta}_q)$ and notice that $\widetilde{\mathcal{V}}(0,\cdot)$ and $\min_{s\in[\delta,1]} \widetilde{\mathcal{V}}(s,\cdot)$ are continuous functions. Thus, there exists $\underline{\theta}_{q_2}$ such that $\widetilde{\mathcal{V}}(0,\underline{\theta}_q) < \min_{s\in[\delta,1]} \widetilde{\mathcal{V}}(s,\underline{\theta}_q)$ for all $\underline{\theta}_q > \underline{\theta}_{q_2}$. Thus, $\widetilde{\mathcal{V}}(0,\underline{\theta}_q) < \widetilde{\mathcal{V}}(s,\underline{\theta}_q)$ for all $s > 0$ and $\underline{\theta}_q > \max\left\{\underline{\theta}_{q_1}, \underline{\theta}_{q_2}\right\}$; that is, $\mathcal{S}^*(\underline{\kappa}(\underline{\theta}_q)) = \{0\}$ for all $\underline{\theta}_q > \max\left\{\underline{\theta}_{q_1}, \underline{\theta}_{q_2}\right\}$.

From (7) and F.3, for $\kappa > \underline{\kappa}(\underline{\theta}_q)$, we have $d\mathcal{V}(0;\kappa,\underline{\theta}_q)/ds < 0$ and hence $0 \notin \mathcal{S}^*(\kappa)$. Analogously, for $\kappa < \overline{\kappa}(\underline{\theta}_q)$, we have $d\mathcal{V}(1;\kappa,\underline{\theta}_q)/ds > 0$ and, hence, $1 \notin \mathcal{S}^*(\kappa)$.

Finally, an argument analogous to that in the corresponding part of the proof of Lemma 3 proves that $\mathcal{S}^*$ is strictly increasing over $\mathcal{S}^{*-1}(0,1) = (\underline{\kappa}(\underline{\theta}_q), \overline{\kappa}(\underline{\theta}_q))$. ■

Proof of Proposition 3. The proof of part (i) is analogous to the proof of Proposition 1, with $\hat{s}(\underline{\theta}_q)$, defined in Lemma 5, playing the role of $\hat{s}_u$:[51] Lemma 5 plays the role of Lemmas 1 and 2, and Lemma 6 plays the role of Lemma 3.

Now we prove part (ii). If F is QS (and hence has a neutral signal), then, by L'Hôpital's rule, $r(s;\underline{\theta}_q) = \left(\frac{\partial f(s,\underline{\theta}_q)}{\partial\theta}\right)^2 \left(\frac{\partial f(s,\theta)}{\partial\theta}\right)^{-2} \frac{C_u''(\theta)}{C_q''(\underline{\theta}_q;\underline{\theta}_q)}$ for $s = 0,1$; and

$$r(s^\star;\underline{\theta}_q) = \frac{F_\theta(s^\star,\theta_q^*(s^\star;\underline{\theta}_q))\frac{\partial^2 f(s^\star,\theta_q^*(s^\star;\underline{\theta}_q))}{\partial\theta\partial s}\left(C_q''(\theta_q^*(s^\star;\underline{\theta}_q)) + \frac{\partial^2 F(s^\star,\theta_q^*(s^\star;\underline{\theta}_q))}{\partial\theta^2}\right)^{-1}}{F_\theta(s^\star,\theta_u^*(s^\star))\frac{\partial^2 f(s^\star,\theta_u^*(s^\star))}{\partial\theta\partial s}\left(C_u''(\theta_u^*(s^\star)) + \frac{\partial^2 F(s^\star,\theta_u^*(s^\star))}{\partial\theta^2}\right)^{-1}}.$$

It follows that r is well defined over $[0,1]$, by Remark 3; furthermore $\max_{[0,1]} r(s;\cdot)$ is continuous and converges to 0 as $\underline{\theta}_q \to \bar{\theta}$. Therefore, for large enough $\underline{\theta}_q$, $r(s;\underline{\theta}_q) < \kappa$ for all $s \in [0,1]$ and $\kappa \geq \underline{\kappa}(\underline{\theta}_q)$; and from part (iii) ((iv)) of Lemma 2, the manager is soft (harsh) in an equilibrium $(s_P^*, \theta_u^*, \theta_q^*)$ if $s_P^* < (>) s^\star$. Thus,

[51] Case 1 in Part 1 in the the proof of Proposition 1 is not necessary here as the assumption $f > 0$ rules out the possibility that $\overline{\kappa}(\underline{\theta}_q) = \infty$ for all $\underline{\theta}_q \in \Theta^\circ$.

there exists $\underline{\theta}_{q_3} < \overline{\theta}$ such that $\hat{s}(\underline{\theta}_q) = s^\star$ for all $\underline{\theta}_q \in (\underline{\theta}_{q_3}, \overline{\theta})$. Further, if F is QS, (i)-(iii) in Claim 2 still hold in the extended model, so the analogous to (iv) in that claim holds as well.[52] Hence, if $\kappa = 1$, then $\mathcal{V}(s) = \mathcal{V}(s_f(s))$ for all $s \in [0,1]$. Therefore $\widetilde{\kappa}(\underline{\theta}_q) := \sup\{\kappa \in (0,\infty) : \sup \mathcal{S}^*(\kappa) \leq \hat{s}(\underline{\theta}_q)\} = 1$ by an argument analogous to that in the proof of Proposition 2. □

Proofs and Ancillary Material of Section 5

Lemma 7 *Assume that F satisfies F.1-F.2 and C_q satisfies C.1 (i)-(iv). In game Γ_3, $\mathcal{S}^*$ is weakly increasing over $(0,\infty)$ and strictly increasing over $(\underline{\kappa}_C, \overline{\kappa}_C)$. Further, $\mathcal{S}^*(\kappa) = \{0\}$ (corresp., $\subset (0,1), = \{1\}$) for all $\kappa \in (0, \underline{\kappa}_C)$ (corresp., $(\underline{\kappa}_C, \overline{\kappa}_C)$, $(\overline{\kappa}_C, \infty)$).*

Proof. The proof that $\mathcal{S}^*$ is weakly increasing is indirect and analogous to the one in the proof of Lemma 3, so we omit it.

By definition, $\underline{\kappa}_C \leq \frac{F(s,\theta_q^*(s))}{F(s,\underline{\theta})}$, which is equivalent to $0 \leq F(s,\theta_q^*(s)) - \underline{\kappa}_C F(s,\underline{\theta})$, for all $s \in (0,1)$. Thus $0 \in \mathcal{S}^*(\underline{\kappa}_C)$ and since $\mathcal{S}^*$ is weakly increasing, $\mathcal{S}^*(\kappa) = \{0\}$ for all $\kappa < \underline{\kappa}_C$. Also by definition, for all $\kappa > \underline{\kappa}_C$, there exists $s \in (0,1)$ such that $\kappa > \frac{F(s,\theta_q^*(s))}{F(s,\underline{\theta})}$, which is equivalent to $0 > F(s,\theta_q^*(s)) - \kappa F(s,\underline{\theta})$, and therefore $0 \notin \mathcal{S}^*(\kappa)$.

An analogous argument shows that $1 \in \mathcal{S}^*(\overline{\kappa}_C)$, $\mathcal{S}^*(\kappa) = \{1\}$ for all $\kappa > \overline{\kappa}_C$ and that $1 \notin \mathcal{S}^*(\kappa)$ for all $\kappa < \overline{\kappa}_C$.

Finally, the argument to prove that $\mathcal{S}^*$ is strictly increasing over $(\underline{\kappa}_C, \overline{\kappa}_C)$ is analogous to the corresponding argument in the proof of Lemma 3. ■

Proof of Proposition 4. First, we establish that $\widetilde{\kappa}_G \in (\underline{\kappa}_C, \overline{\kappa}_C)$. Observe that $V(\hat{s}_q, \underline{\theta}, \theta_q^*(\hat{s}_q); \widetilde{\kappa}_G) < V(s, \underline{\theta}, \theta_q^*(\hat{s}_q); \widetilde{\kappa}_G) < V(s, \underline{\theta}, \theta_q^*(s); \widetilde{\kappa}_G)$, for all $s \in [0,1] \setminus \{\hat{s}_q\}$, where the first inequality follows from the fact that $\hat{s}_q$ is the unique minimizer of $V(\cdot, \underline{\theta}, \theta_q^*(\hat{s}_q); \widetilde{\kappa}_G)$,[53] and the second inequality follows from observing that $V(s, \underline{\theta}, \cdot; \widetilde{\kappa}_G)$ is decreasing for all $s \in (0,1)$ and $\hat{s}_q$ is the unique maximizer of θ_q^*. Thus, $\mathcal{S}^*(\widetilde{\kappa}_G) = \{\hat{s}_q\}$, and since in the proof of Lemma 7 it is shown that $\mathcal{S}^*$ is strictly increasing over $(\underline{\kappa}_C, \overline{\kappa}_C)$, $0 \in \mathcal{S}^*(\underline{\kappa}_C)$ and $1 \in \mathcal{S}^*(\overline{\kappa}_C)$, we conclude $\widetilde{\kappa}_G \in (\underline{\kappa}_C, \overline{\kappa}_C)$.

Now we prove that the manager is ex-post efficient for all $\kappa \in (0, \underline{\kappa}_C)$. Notice that $\underline{\kappa}_C \leq \lim_{s\to 0} \frac{F(s,\theta_q^*(s))}{F(s,\underline{\theta})} = \lim_{s\to 0} \frac{d_q(s)}{f(s,\underline{\theta})} \leq \lim_{s\to 0} g(s, \underline{\theta}, \theta_q^*(s)) = g(0, \underline{\theta}, \underline{\theta}_q)$. Thus, $\underline{\kappa}_C \leq g(0, \underline{\theta}, \underline{\theta}_q)$. The ex-post optimal standard is weakly increasing in κ

[52] In particular, part (ii) of Claim 2 holds for both θ_q^* and θ_u^*.

[53] Notice that by definition of $\widetilde{\kappa}_G$, $\hat{s}_q$ is the ex-post optimal standard if fit candidates' readiness is $\theta_q^*(\hat{s}_q)$, unfit candidates' readiness is $\underline{\theta}$, and $\kappa = \widetilde{\kappa}_G$.

and $s = 0$ is the only ex-post optimal standard for $\kappa = g(0, \underline{\theta}, \underline{\theta}_q)$ (see Section 2.2). Thus, since $\underline{\kappa}_C \leq g(0, \underline{\theta}, \underline{\theta}_q)$ and $\mathcal{S}^*(\kappa) = 0$ for all $\kappa < \underline{\kappa}_C$ (Lemma 7), we have that the manager is ex-post efficient for all $\kappa < \underline{\kappa}_C$.

The argument proving that the manager is harsh for all $\kappa \in (\underline{\kappa}_C, \widetilde{\kappa}_G)$ is analogous to the argument showing that the manager is harsh for all $\kappa \in (\widetilde{\kappa}_U, \overline{\kappa})$ in the proof of Proposition 1. Instead of Lemma 3, we use Lemma 7.

The arguments showing that the manager is ex-post efficient for all $\kappa \in (\overline{\kappa}_C, \infty)$ and soft for all $\kappa \in (\widetilde{\kappa}_G, \overline{\kappa}_C)$ are analogous to the arguments showing that the manager is ex-post efficient for all $\kappa \in (0, \underline{\kappa}_C)$ and harsh for all $\kappa \in (\underline{\kappa}_C, \widetilde{\kappa}_G)$, respectively. Finally, by (7), if $\kappa = \widetilde{\kappa}_G$, the manager is ex-post efficient. □

Proof of Proposition 5: In the sequel, when convenient, we make explicit the dependence of d_u, r, and v on the unfit candidates' cost parameter λ, so, instead of writing $d_u(s)$, $r(s)$, and $v(s)$, we write $d_u(s; \lambda)$, $r(s; \lambda)$, and $v(s; \lambda)$, respectively, for all $s \in [0, 1]$ and $\lambda \in [0, 1]$. The proof hinges on the following lemmata:

Lemma 8 *Assume F.2-F.3 and C.1-C.2. Then, there exists $\overline{\lambda}_1 > 0$ such that for all $\lambda \in [0, \overline{\lambda}_1)$, $d_u(s; \lambda) > 0$ for all $s \in [0, 1]$.*

Proof. The Maximun Theorem and F.3 guarantee that $\min_{s\in[0,1]} d_u(s; \lambda)$ varies continuously with λ. Further, $\min_{s\in[0,1]} d_u(s; \lambda) = \min_{s\in[0,1]} f(s, \underline{\theta}) > 0$ for $\lambda = 0$. Thus, there exists $\overline{\lambda}_1 > 0$ such that $\min_{s\in[0,1]} d_u(s; \lambda) > 0$ for all $\lambda \in [0, \overline{\lambda}_1)$. ■

Lemma 9 *Assume F.2-F.3 and C.1-C.2. Then, there exists $\overline{\lambda}_2 > 0$ such that for all $\lambda \in \left[0, \overline{\lambda}_2\right)$, there exists $\hat{s}(\lambda) \in (0, 1)$ such that if $(s_P^*, \theta_u^*(\cdot; \lambda), \theta_q^*)$ is a SPNE of $\Gamma_1(\lambda)$, then the manager is harsh if $s_P^* \in (0, \hat{s}(\lambda))$ and soft if $s_P^* \in (\hat{s}(\lambda), 1)$.*

Proof. The proof is analogous to the proof of Lemma 5, using the fact that $v(\cdot; \lambda)$ approaches to $v(\cdot; 0)$ (instead of $v(\cdot; \underline{\theta}_q)$ approaches to $v(\cdot; \overline{\theta})$) and $g(\cdot, \theta_u^*(\cdot; \lambda), \theta_q^*(\cdot))$ approaches to $g(\cdot, \underline{\theta}, \theta_q^*(\cdot))$ (instead of $g(\cdot, \theta_u^*(\cdot), \theta_q^*(\cdot; \underline{\theta}_q))$ approaches to $g(\cdot, \theta_u^*(\cdot), \overline{\theta})$), as $\lambda \to 0$ (instead of as $\underline{\theta}_q \to \overline{\theta}$). ■

Lemma 10 *Assume F.2-F.3 and C.1-C.2. Then, there exists $\overline{\lambda}_3 > 0$ such that for all $\lambda \in \left[0, \overline{\lambda}_3\right)$, the thesis of Lemma 7 holds in $\Gamma_1(\lambda)$, mutatis mutandis, replacing $\underline{\kappa}_C$ and $\overline{\kappa}_C$ with $\underline{\kappa}_C(\lambda)$ and $\overline{\kappa}_C(\lambda)$, respectively.*

Proof. From Lemma 8, $F(\cdot, \theta_u^*(\cdot; \lambda))$ is strictly increasing for all $\lambda < \overline{\lambda}_1$. Then, an argument analogous to the one in the proof of Lemma 7 applies with $\underline{\kappa}_C(\lambda)$ and $\overline{\kappa}_C(\lambda)$ playing the role of $\underline{\kappa}_C$ and $\overline{\kappa}_C$, respectively. ■

Proof of Proposition 5. Let $\widetilde{\kappa}_G(\lambda) := g(\hat{s}(\lambda), \theta_u^*(\hat{s}(\lambda); \lambda), \theta_q^*(\hat{s}(\lambda)))$. Since $\lim_{\lambda\to 0} \hat{s}(\lambda) = \hat{s}_q$, we have $\lim_{\lambda\to 0} \widetilde{\kappa}_G(\lambda) = \widetilde{\kappa}_G$. Similarly, $\lim_{\lambda\to 0} \underline{\kappa}_C(\lambda) = \underline{\kappa}_C$ and

$\lim_{\lambda \to 0} \overline{\kappa}_C(\lambda) = \overline{\kappa}_C$. Thus, there exists $\overline{\lambda}_4 > 0$ such that $\underline{\kappa}_C(\lambda) < \widetilde{\kappa}_G(\lambda) < \overline{\kappa}_C(\lambda)$ for all $\lambda \in (0, \overline{\lambda}_4)$.

For part (i), the above lemmata allow us to provide an argument analogous to the one in the proof of Proposition 4, with Lemma 9 playing the role of Lemma 1 and 2, and Lemma 10 playing the role of Lemma 7. We omit the details.

The proof of part (ii) is analogous to the proof of part (ii) of Proposition 3, but showing that $\min_{s \in [0,1]} r(s; \lambda)$ goes to ∞ as $\lambda \to 0$, which implies that there exists $\overline{\lambda}_5 > 0$ such that $r(s) > \kappa$ for all $s \in [0, 1]$, $\kappa < \overline{\kappa}_C(\lambda)$ and $\lambda < \overline{\lambda}_5$. The rest of the argument is analogous and the details are omitted. Indeed, since in this case, $\mathcal{S}^*(1) = \{s^\star\}$, the argument is slightly more direct. □

Proofs and Ancillary Material of Section 6

Proof of Corollary 4. This result is a consequence of Proposition 3. Take $\underline{\theta}_q$ to be the same as in Proposition 3. Consider an arbitrary initial advantage $\underline{\theta}_q > \underline{\theta}_q$ and prior $\kappa \in (\underline{\kappa}(\underline{\theta}_q), \widetilde{\kappa}(\underline{\theta}_q))$. From Proposition 3, if $(s_P^*, \theta_u^*, \theta_q^*(\cdot; \underline{\theta}_q))$ is a SPNE of $\Gamma_1(\underline{\theta}_q)$, then the manager is soft at that equilibrium, and thus, $\kappa = v(s_P^*; \underline{\theta}_q) > g(s_P^*, \theta_u^*(s_P^*), \theta_q^*(s_P^*; \underline{\theta}_q))$. Since $g(1, \theta_u^*(1), \theta_q^*(1; \underline{\theta}_q)) = \overline{\kappa}(\underline{\theta}_q) > \widetilde{\kappa}(\underline{\theta}_q) > \kappa$, there exists $s \in (s_P^*, 1)$ such that $g(s, \theta_u^*(s), \theta_q^*(s; \underline{\theta}_q)) = \kappa$, by the Intermediate Value Theorem. Hence, $(s, \theta_u^*(s), \theta_q^*(s; \underline{\theta}_q))$ is a BNE of $\Gamma_0(\underline{\theta}_q)$. This proves the first part.

For the uniqueness, note that $\frac{dg(s, \theta_u^*(s), \theta_q^*(s; \underline{\theta}_q))}{ds} = g(s, \theta_u^*(s), \theta_q^*(s; \underline{\theta}_q))\varphi(s; \underline{\theta}_q)$, where

$$\begin{aligned} \varphi(s; \underline{\theta}_q) :=& \frac{1}{f(s, \theta_q^*(s; \underline{\theta}_q))} \frac{\partial f(s, \theta_q^*(s; \underline{\theta}_q))}{\partial s} - \frac{1}{f(s, \theta_u^*(s))} \frac{\partial f(s, \theta_u^*(s))}{\partial s} \\ &+ m(s, \theta_q^*(s; \underline{\theta}_q)) \frac{d\theta_q^*(s; \underline{\theta}_q)}{ds} - m(s, \theta_u^*(s)) \frac{d\theta_u^*(s)}{ds}, \end{aligned} \tag{17}$$

for all $s \in [0, 1]$. Thus, $\frac{dg(s, \theta_u^*(s), \theta_q^*(s; \underline{\theta}_q))}{ds} > 0$ if and only if $\varphi(s; \underline{\theta}_q) > 0$. The first line on the right hand side of (17) is strictly positive for all $s \in [0, 1]$, due to Assumption F.2, the hypothesis, and the fact that $\theta_q^* > \theta_u^*$.

In addition, $F_\theta(\cdot, \bar{\theta}) = 0$ implies $\lim_{\underline{\theta}_q \to \bar{\theta}} m(s, \theta_q^*(s; \underline{\theta}_q)) \frac{d\theta_q^*(s; \underline{\theta}_q)}{ds} = 0$. Since $\varphi(s; \cdot)$ is continuous, using the Maximum Theorem, we have $\lim_{\underline{\theta}_q \to \bar{\theta}} \min_{s \in [0,1]} \varphi(s; \underline{\theta}_q) > 0$. This yields the result. □

Proof of Corollary 5. This result is essentially a consequence of Proposition 5. Consider $\overline{\lambda}$ as in Proposition 5 and arbitrary $\lambda < \overline{\lambda}$ and prior $\kappa \in (\widetilde{\kappa}_G(\lambda), \overline{\kappa}_C(\lambda))$. From Proposition 5, if $(s_P^*, \theta_u^*(\cdot; \lambda), \theta_q^*)$ is a SPNE of $\Gamma_1(\lambda)$, then the manager is soft at that equilibrium, and thus, $\kappa = v(s_P^*; \lambda) > g(s_P^*, \theta_u^*(s_P^*; \lambda), \theta_q^*(s_P^*))$.

If $\kappa \leq \max_{[s_P^*,1]} \left\{g(s, \theta_u^*(s;\lambda), \theta_q^*(s))\right\}$, then there exists $s \in (s_P^*, 1]$ such that $\kappa = g(s, \theta_u^*(s;\lambda), \theta_q^*(s))$. Thus, $(s, \theta_u^*(s;\lambda), \theta_q^*(s))$ is a BNE of $\Gamma_0(\lambda)$. This proves the result for this case.

If $\kappa > \max_{[s_P^*,1]} \left\{g(s, \theta_u^*(s;\lambda), \theta_q^*(s))\right\}$, then $\kappa > g(1, \theta_u^*(1;\lambda), \theta_q^*(1))$, and thus, $(1, \underline{\theta}, \underline{\theta}_q)$ is a BNE of $\Gamma_0(\lambda)$. This proves the result for this case.

Finally uniqueness, as in the proof of Corollary 4, follows from the fact that for high enough $\underline{\theta}_q$, the minimum with respect to s of the sum of the first three terms on the right-hand side of (17) is positive,[54] an so it is the last term. □

Proofs and Ancillary Material of Section 7

Optimality of (s,p) mechanisms. We now show that allowing for positive probabilities that (i) a candidate reporting to be unfit is tested, and (ii) a candidate reporting to be fit is outright hired or outright rejected, cannot decrease the manager's expected loss beyond what he can attain within the (s,p) class:

(i) Any mechanism that, with a strictly positive probability, asks a candidate reporting to be unfit to take a test with standard s can be improved by other mechanism that increases the probability of outright rejection of that candidate by $F(s, \theta_u^*(s)) + C_u(\theta_u^*(s))$ times the probability that she is subjected to the test in the former mechanism. Such a change would not affect the unfit candidate's expected payoff from reporting unfit and would decrease the fit candidate's expected payoff from reporting unfit, because $F(s, \theta_u^*(s)) + C_u(\theta_u^*(s)) > F(s, \theta_q^*(s)) + C_q(\theta_q^*(s))$. That is, incentive compatibility would still hold. Finally, the manager would be strictly better-off due to the higher probability of rejecting the unfit candidate.

(ii) Now we show that allowing for a strictly positive probability of outright rejection or outright hiring of a candidate reporting to be fit cannot make the manager better-off. Since the incentive compatibility constraint for the unfit candidate is binding, the probability of rejecting a candidate who claims to be unfit is $p = p_1 + (1 - p_1 - p_2)\left(F(s, \theta_u^*(s)) + C_u(\theta_u^*(s))\right)$, where p_1 (p_2) is the probability of outright rejecting (hiring) a candidate reporting to be fit, and s is the standard of the test. Hence, the manager's expected loss is an affine transformation of

$$p_1 + (1-p_1-p_2)F(s, \theta_q^*(s)) - \kappa p = p_1 V_M(1) + p_2 V_M(0) + (1-p_1-p_2)V_M(s), \quad (18)$$

for all (s, p_1, p_2) with $s \in [0,1]$ and $p_1, p_2, 1 - p_1 - p_2 \geq 0$. An argument parallel to the one used in the proof of Lemma 7 proves that 0 is the unique minimizer

[54] The sum of the first three terms is greater than in the proof of Corollary 4 now that $\theta_u^*(\cdot;1)$ is replaced by $\theta_u^*(\cdot;\lambda)$ with $\lambda < 1$.

(corresp., is a minimizer, is not a minimizer) of V_M for all $\kappa < \underline{\kappa}_M$ (corresp., for $\kappa = \underline{\kappa}_M$, for all $\kappa > \underline{\kappa}_M$). Similarly, 1 is the unique minimizer (corresp., is a minimizer, is not a minimizer) of V_M for all $\kappa > \overline{\kappa}_M$ (corresp., for $\kappa = \overline{\kappa}_M$, for all $\kappa < \overline{\kappa}_M$). Thus, (18) implies that the mechanism $(s_M, p_M) = (0, 0)$ is optimal for all $\kappa \leq \underline{\kappa}_M$, $(s_M, p_M) = (1, 1)$ is optimal for all $\kappa \geq \overline{\kappa}_M$, and, for all $\kappa \in (\underline{\kappa}_M, \overline{\kappa}_M)$, there exists $s \in (0, 1)$ such that $(s_M, p_M) = (s, F(s, \theta_u^*(s)) + C_u(\theta_u^*(s)))$ is optimal.

Proof of Proposition 6. We proved part (i) in the previous paragraph. For part (ii), we have that $V_M(s) < \mathcal{V}(s)$ for all $s \in (0, 1)$. Notice that $\underline{\kappa}_M \leq \underline{\kappa}_C(1) < \overline{\kappa}_C(1) \leq \overline{\kappa}_M$. An argument parallel to the one in the proof of Lemma 7 reveals that, in Γ_1, 0 is the unique minimizer (corresp., is a minimizer, is not a minimizer) of $\mathcal{V}$ for all $\kappa < \underline{\kappa}_C(1)$ (corresp., for $\kappa = \underline{\kappa}_C(1)$, for all $\kappa > \underline{\kappa}_C(1)$). Similarly, 1 is the unique minimizer (corresp., is a minimizer, is not a minimizer) of $\mathcal{V}$ for all $\kappa > \overline{\kappa}_C(1)$ (corresp., for $\kappa = \overline{\kappa}_C(1)$, for all $\kappa < \overline{\kappa}_C(1)$). Thus, $\min_{s\in[0,1]} V_M(s) < \min_{s\in[0,1]} \mathcal{V}(s)$ for all $\kappa \in (\underline{\kappa}_M, \overline{\kappa}_M)$.

Finally, we establish the increasingness of $\mathcal{S}_M^*$. Consider $\kappa' > \kappa$, $s \in \mathcal{S}_M^*(\kappa)$, and $s' \in \mathcal{S}_M^*(\kappa')$. Then,

$$F(s, \theta_q^*(s)) - \kappa\, (F(s, \theta_u^*(s)) + C_u(\theta_u^*(s))) \leq F(s', \theta_q^*(s')) - \kappa \left(F(s', \theta_u^*(s')) + C_u(\theta_u^*(s'))\right)$$
$$F(s', \theta_q^*(s')) - \kappa' \left(F(s', \theta_u^*(s')) + C_u(\theta_u^*(s'))\right) \leq F(s, \theta_q^*(s)) - \kappa' \left(F(s, \theta_u^*(s)) + C_u(\theta_u^*(s))\right).$$

Adding these inequalities yields

$$(\kappa' - \kappa)\left[(F(s, \theta_u^*(s)) + C_u(\theta_u^*(s))) - (F(s', \theta_u^*(s')) + C_u(\theta_u^*(s')))\right] \leq 0,$$

which implies $F(s, \theta_u^*(s)) + C_u(\theta_u^*(s)) \leq F(s', \theta_u^*(s')) + C_u(\theta_u^*(s'))$. By the Envelope Theorem, $F(\cdot, \theta_u^*(\cdot)) + C_u(\theta_u^*(\cdot))$ is strictly increasing; thus, $s \leq s'$. $\square$

Proof of Proposition 7. Let $v(\cdot; \underline{\theta}_q, \lambda) := d_q(\cdot; \underline{\theta}_q)/d_u(\cdot; \lambda)$, for all $\underline{\theta}_q \in \Theta^\circ$ and $\lambda \in (0, 1)$. We denote explicitly the dependence dependence of $\underline{\kappa}_C(\lambda)$ and $\overline{\kappa}_C(\lambda)$ on $\underline{\theta}_q$, writing $\underline{\kappa}_C(\underline{\theta}_q, \lambda)$ and $\overline{\kappa}_C(\underline{\theta}_q, \lambda)$, respectively, for all $\underline{\theta}_q \in \Theta^\circ$ and $\lambda \in (0, 1)$.

By Assumption F.2 and the Maximum Theorem,

$$\lim_{(\underline{\theta}_q, \lambda) \to (\overline{\theta}, 0)} \min_{s\in[0,1]} \frac{dv(s; \underline{\theta}_q, \lambda)}{ds} = \lim_{(\underline{\theta}_q, \lambda) \to (\overline{\theta}, 0)} \min_{s\in[0,1]} \frac{dv_M(s; \theta_u^*(s; \lambda))}{ds} = \min_{s\in[0,1]} \frac{dg(s, \underline{\theta}, \overline{\theta})}{ds} > 0.$$

Thus, there exist $\underline{\underline{\theta}_q}$ and $\overline{\lambda}$ such that $v(\cdot; \underline{\theta}_q, \lambda)$ and $v_M(\cdot; \theta_u^*(\cdot; \lambda))$ are strictly increasing for all $(\underline{\theta}_q, \lambda)$ such that $\underline{\theta}_q > \underline{\underline{\theta}_q}$ and $\lambda < \overline{\lambda}$. Further, $v(s; \underline{\theta}_q, \lambda) = v_M(s; \theta_u^*(s; \lambda))$ at $s = 0, 1$.

Suppose $\underline{\theta}_q > \underline{\underline{\theta}_q}$ and $\lambda < \overline{\lambda}$. For all $\kappa \leq \underline{\kappa}_M(\underline{\theta}_q, \lambda)$, $s_M = 0$ and for all

$\kappa \geq \overline{\kappa}_M(\underline{\theta}_q, \lambda)$, $s_M = 1$. Similarly, for all $\kappa \leq \underline{\kappa}_C(\underline{\theta}_q, \lambda)$, $s_P^* = 0$ and for all $\kappa \geq \overline{\kappa}_C(\underline{\theta}_q, \lambda)$, $s_P^* = 1$. Furthermore, for all $\kappa \in (\underline{\kappa}_M(\underline{\theta}_q, \lambda), \overline{\kappa}_M(\underline{\theta}_q, \lambda))$, s_M is the root of $v_M(\cdot; \theta_u^*(\cdot; \lambda)) = \kappa$. Similarly, for all $\kappa \in (\underline{\kappa}_C(\underline{\theta}_q, \lambda), \overline{\kappa}_C(\underline{\theta}_q, \lambda))$, s_P^* is the root of $v(\cdot; \underline{\theta}_q, \lambda) = \kappa$. Since $\underline{\kappa}_M(\underline{\theta}_q, \lambda) \leq \underline{\kappa}_C(\underline{\theta}_q, \lambda) < \overline{\kappa}_C(\underline{\theta}_q, \lambda) \leq \overline{\kappa}_M(\underline{\theta}_q, \lambda)$, and over $(0, 1)$, $v(s; \underline{\theta}_q, \lambda) > (=, <) v_M(s; \theta_u^*(s; \lambda))$ if $s < (=, >) \hat{s}_u(\lambda)$, we conclude $s_P^* < (>)s_M$ if and only if $\kappa \in (\underline{\kappa}_M(\underline{\theta}_q, \lambda), v(\hat{s}_u(\lambda); \underline{\theta}_q, \lambda))$ $(\kappa \in (v(\hat{s}_u(\lambda); \underline{\theta}_q, \lambda), \overline{\kappa}_M(\underline{\theta}_q, \lambda)))$.
□

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